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One-loop leading logarithms in electroweak radiative corrections

II. Factorization of collinear singularities

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Abstract. We discuss the evaluation of the collinear single-logarithmic contributions to virtual electroweak corrections at high energies. More precisely, we prove the factorization of the mass singularities originating from loop diagrams involving collinear virtual gauge bosons coupled to external legs. We discuss, in particular, processes involving external longitudinal gauge bosons, which are treated using the Goldstone-boson equivalence theorem. The proof of factorization is performed within the 't Hooft–Feynman gauge at one-loop order and applies to arbitrary electroweak processes that are not mass-suppressed at high energies. As basic ingredients we use Ward identities for Green functions with arbitrary external particles involving a gauge boson collinear to one of these. The Ward identities are derived from the BRS invariance of the spontaneously broken electroweak gauge theory.

1 Introduction

In the energy range above the electroweak scale, $s^{1/2} \gg M_W$, electroweak radiative corrections are dominated by double-logarithmic (DL) terms of the form $\alpha \log^2 (s/M_W^2)$ and single-logarithmic (SL) terms of the form $\alpha \log(s/M_W^2)$ involving the ratio of the energy to the electroweak scale (see [1–8] and references therein). Such corrections grow with energy, and at $s^{1/2} = 0.5$–$1\,\mathrm{TeV}$ they are typically of order 10% of the theoretical prediction. In the TeV range, the SL terms are numerically of the same size as the DL terms.

For electroweak processes that are not mass-suppressed at high energies, these leading logarithmic corrections are universal. On the one hand, single logarithms originating from short-distance scales result from the renormalization of dimensionless parameters, i.e. the running of the gauge, Yukawa, and scalar couplings. On the other hand, universal logarithms originating from the long-distance scale $M_W \ll s^{1/2}$ are expected to factorize, i.e. they can be associated with external lines or pairs of external lines in Feynman diagrams. They consist of DL and SL terms originating from soft-collinear and collinear (or soft) gauge bosons, respectively, coupled to external legs. The non-logarithmic terms are in general non-universal and have to be evaluated for each process separately if needed.

In the recent literature (see [1–8] and references therein), most interest has been devoted to electroweak long-distance corrections, which have often been compared to the well-known soft and collinear singularities observed in QCD (see for instance [9]). This is a useful guideline in order to understand universal effects, and also to discuss specific features that distinguish a spontaneously broken gauge theory from a symmetric one.

The main difference between QCD and the electroweak standard model is that the masses of the weak gauge bosons provide a physical cut-off for real Z- and W-boson emission. Therefore, for a sufficiently good experimental resolution, soft and collinear weak-boson radiation need not be included in the theoretical predictions and, except for electromagnetic real corrections, we can restrict ourselves to large logarithms originating from virtual corrections.

Here we concentrate on the factorization of virtual collinear corrections in high-energy electroweak reactions. In QCD, factorization is strictly connected to gauge symmetry [9]. Therefore, it is natural to ask if and how factorization is affected by the spontaneous breaking of the gauge symmetry within the electroweak theory.

In the literature [10], this question has been avoided by assuming that "the electroweak theory is in the symmetric phase at high energies". In this case, one restricts oneself to the symmetric part of the electroweak Lagrangian ($\mathcal{L}_{\mathrm{symm}}$), which corresponds to a vanishing vacuum expectation value (vev) of the scalar doublet and depends only on dimensionless parameters; gauge-boson masses in the propagators act merely as infrared cut-off. In this "symmetric approach", methods and results obtained within QCD are extended to the electroweak theory [3,4,10]. Un-

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der these assumptions, only the following specifically electroweak ingredients need to be included:

(1) Yukawa and scalar sector: since the dimensionless Yukawa and scalar couplings are proportional to the fermion and Higgs-boson masses, respectively, their effects are enhanced if these particles are heavy. Especially, one finds large logarithmic corrections proportional to $m_t^2/M_W^2 \log(s/M_W^2)$ for processes involving heavy quarks or Higgs bosons.
(2) Mixing of neutral gauge bosons: the neutral mass-eigenstate gauge bosons A and Z originate from mixing between the U(1) and SU(2) eigenstates. Since the adjoint representation of the SU(2)×U(1) group is not irreducible, factorization is non-diagonal for processes involving external photons and Z bosons. Note that the definition of the mass eigenstates requires one to consider the theory at the electroweak scale.

The symmetric approach seems to be adequate for electroweak processes involving only fermions, transverse gauge bosons, and Higgs bosons as external particles, since these states are already present in the symmetric phase. However, it is less clear whether this approach is adequate for processes involving longitudinal gauge bosons, which originate from spontaneous symmetry breaking.

For a rigorous treatment and, in particular, for processes involving arbitrary external fields corresponding to mass eigenstates of the electroweak theory, we need a "complete electroweak" approach. Therefore, we calculate the leading logarithmic one-loop corrections that originate from the complete Lagrangian, including terms proportional to the vev. However, we restrict ourselves to processes that are not mass-suppressed in the high-energy limit, i.e. processes originating from $\mathcal{L}_{\mathrm{symm}}$ in lowest order. A process is called mass-suppressed if its matrix element with mass dimension d does not scale as E^d in the high-energy limit $E \gg M_W$ but with $E^{d-n}M_W^n$, $n>0$. To prove the factorization of the virtual collinear single logarithms, we use Ward identities that are based on the symmetry of the complete Lagrangian.

In particular, we discuss the effects that are related to the part of the Lagrangian that results from spontaneous symmetry breaking ($\mathcal{L}_v$), i.e. the part proportional to the non-vanishing vev. The part $\mathcal{L}_v$ consists of terms that are bilinear and trilinear in the fields. In lowest order, bilinear terms in the scalar sector provide gauge-boson masses and mixing between gauge bosons and would-be Goldstone bosons. Corresponding mixing terms are introduced in the 't Hooft gauge-fixing Lagrangian. As a consequence of the BRS invariance, the mixing terms lead to the well-known Goldstone-boson equivalence theorem (GBET) [11], which relates longitudinal gauge bosons to would-be Goldstone bosons in the high-energy limit. Beyond tree level, also the trilinear couplings with mass dimension in $\mathcal{L}_v$ have to be taken into account, since they give leading SL corrections to the mass and mixing terms, and thus corrections to the GBET (for the corrections to the GBET see [12]).

The complete one-loop results for high-energy leading electroweak DL and SL corrections have been presented in [7]. They include soft-collinear, purely collinear, purely soft, as well as parameter-renormalization contributions. In this article we concentrate on the purely collinear SL corrections. Especially, we prove the non-trivial factorization of the part originating from mass-singular loop diagrams in the 't Hooft–Feynman gauge. In Sect. 2 (and Appendix A) we discuss mass singularities originating from loop diagrams and show that they are restricted to virtual gauge bosons coupled to external lines. The factorization of these mass singularities is demonstrated in Sect. 3 using *collinear Ward identities*. We also recall the complete gauge-invariant results for the collinear and soft single-logarithmic corrections given in [7], including the part originating from renormalization (field-renormalization constants and corrections to the GBET). The collinear Ward identities, which constitute the basis for the proof, are derived in Sect. 4 using the BRS invariance of the electroweak theory (Appendix B). Our conventions for Green functions can be found in Appendix C.

2 Collinear mass singularities

2.1 Notation

We consider electroweak processes involving n arbitrary external particles. Lowest-order (LO) matrix elements are denoted by

$$\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i_n}}(p_1,\ldots,p_n), \tag{2.1}$$

where all momenta are considered to be incoming. The (incoming) fields φ_{i_k} represent physical fields in the standard model, i.e. fields corresponding to mass eigenstates for fermions, gauge bosons, or Higgs bosons. Longitudinal gauge bosons are replaced by the corresponding would-be Goldstone bosons via the Goldstone-boson equivalence theorem (GBET). In the limit where all external momenta p_k are on-shell, and all other invariants are much larger than the gauge-boson masses, i.e.

$$\left(\sum_{l=1}^{N} p_{k_l}\right)^2 \sim s \gg M_W^2, \tag{2.2}$$

with $1<N<n-1$ and $k_l \neq k_{l'}$ for $l \neq l'$, the one-loop corrections to (2.1) receive large mass-singular logarithmic contributions. Here, we assume that all invariants are of the order s, the square of the typical energy scale of the considered process, and we restrict ourselves to purely collinear contributions containing terms of the form $\alpha\log(s/M^2)$, where M is equal to M_W or to a light-fermion mass. We show that these corrections $\delta^{\mathrm{C}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}}$ factorize and can be associated to the external states,

$$\delta^{\mathrm{C}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_{k=1}^{n}\sum_{\varphi_{i'_k}} \delta^{\mathrm{C}}_{\varphi_{i'_k}\varphi_{i_k}} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}. \tag{2.3}$$

This universal (process-independent) result has been obtained within the 't Hooft–Feynman gauge, using the independence of the S matrix of the scale μ of dimensional regularization [7]. For external fermions, transverse gauge bosons, and Higgs bosons, the large logarithms are isolated in the μ-dependent part of field-renormalization constants (FRC's) δZ and universal collinear factors δ^{coll} from mass-singular loop diagrams,

$$\delta^{\mathrm{C}}_{\varphi_{i'_k}\varphi_{i_k}} = \left(\frac{1}{2}\delta Z_{\varphi_{i'_k}\varphi_{i_k}} + \delta^{\mathrm{coll}}_{\varphi_{i'_k}\varphi_{i_k}}\right)\Big|_{\mu^2=s}. \tag{2.4}$$

External longitudinal gauge bosons $V^b_{\mathrm{L}} = Z_{\mathrm{L}}, W^{\pm}_{\mathrm{L}}$ are related to the corresponding would-be Goldstone bosons $\Phi_b = \chi, \phi^{\pm}$ using the GBET. The corresponding collinear corrections are given by

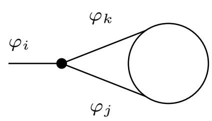
$$\delta^{\mathrm{C}}_{V^{b'}_{\mathrm{L}}V^b_{\mathrm{L}}} = \left(\delta_{V^{b'}V^b}\delta C_{\Phi_b} + \delta^{\mathrm{coll}}_{\Phi_{b'}\Phi_b}\right)\Big|_{\mu^2=s}, \tag{2.5}$$

and depend on the collinear factors for would-be Goldstone bosons and on the corrections δC_{Φ_b} to the GBET. These latter contain the FRC's for gauge bosons, longitudinal self-energy and mixing-energy contributions, and mass counterterms [7].

The FRC's and the corrections to the GBET factorize in an obvious way. Explicit results for these contributions have been presented in [7]. In the following, we discuss only the non-trivial factorization of mass-singular truncated loop diagrams leading to the collinear factors δ^{coll}.

2.2 Mass singularities in loop diagrams

As has been proved by Kinoshita [13], mass-singular logarithmic corrections arise from loop diagrams where an external on-shell line splits into two collinear internal lines,



. (2.6)

Here and in the following, all on-shell external lines that are not involved in our argumentation are omitted in the graphical representation. The diagrams have to be understood as truncated; the self-energy insertions in external legs and the corresponding mass singularities enter the FRC's in (2.4).

We consider splittings $\varphi_i(p) \to \varphi_j(q)\varphi_k(p-q)$ involving arbitrary combinations of fields. These lead to loop integrals of the type

$$\begin{aligned} I = {} & -\mathrm{i}(4\pi)^2\mu^{4-D} \\ & \times \int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{N_{ijk}(q)}{(q^2-M_j^2+\mathrm{i}\varepsilon)[(p-q)^2-M_k^2+\mathrm{i}\varepsilon]}. \end{aligned} \tag{2.7}$$

The part denoted by $N_{ijk}(q)$ is kept implicit. It consists of the LO contribution from the "white blob" in (2.6), of the wave function (spinor or polarization vector) corresponding to the external line φ_i, of the $\varphi_i\varphi_j\varphi_k$ vertex, and of the numerators of the φ_j and φ_k propagators. Since the soft contributions can be treated in the eikonal approximation [7], we assume that the part of $N_{ijk}(q)$ that is singular in the soft limits $q^\mu \to 0$ and $q^\mu \to p^\mu$ has been subtracted (see Sect. 3).

The mass singularity in (2.7) originates from the denominators of the φ_j and φ_k propagators in the collinear region $q^\mu \to xp^\mu$. This is discussed in detail in Appendix A, where we show that the mass singularity can be extracted from (2.7) by treating the integrand $N_{ijk}(q)$ in the collinear approximation (A.11). The resulting contribution reads

$$I \overset{\mathrm{LA}}{=} \log\left(\frac{\mu^2}{M^2}\right)\int_0^1 \mathrm{d}x N_{ijk}(xp) \tag{2.8}$$

in logarithmic approximation (LA), where $M^2 \sim \max(p^2, M_j^2, M_k^2)$. Since we consider all masses M_W, M_Z, M_H, and m_t to be of the same order of magnitude, the scale M is either given by M_W or by a light-fermion mass.

If we now apply the collinear approximation (A.11) to all splittings $\varphi_i \to \varphi_j\varphi_k$, which are allowed by the electroweak Feynman rules [14], it turns out that N_{ijk} is mass-suppressed in most of the cases. This can be easily verified by looking at the external part of the diagram (2.6), containing the φ_i wave function, the $\varphi_i\varphi_j\varphi_k$ vertex, and the numerators of the φ_j and φ_k propagators. Many contributions are proportional to M, p^2, $p_\mu\varepsilon^\mu(p)$, or $\not{p}u(p)$ and thus mass-suppressed. Consider as an example the case $V_{\mathrm{T}} \to \Psi\bar{\Psi}$, where a transverse gauge boson splits into a fermion–antifermion pair. Here

$$\begin{aligned} N(q) & \propto \varepsilon^\mu_{\mathrm{T}}(p)(\not{p}-\not{q})\gamma_\mu\not{q} \\ & \longrightarrow x(1-x)\varepsilon^\mu_{\mathrm{T}}(p)(2p_\mu\not{p} - p^2\gamma_\mu) \end{aligned} \tag{2.9}$$

is mass-suppressed in the collinear limit, $q^\mu \to xp^\mu$, owing to $p_\mu\varepsilon^\mu_{\mathrm{T}}(p) = 0$ and $p^2 \ll s$. Similar suppressions occur in all cases, except for the splittings $\varphi_i \to V^a\varphi_{i'}$ where a virtual gauge boson $V^a = A, Z, W^{\pm}$ is emitted and φ_i and $\varphi_{i'}$ are both fermions, gauge bosons, or scalars. These unsuppressed splittings are considered in Sect. 3.

3 Factorization of collinear singularities

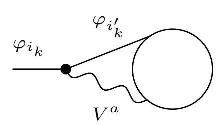
In this section, we evaluate the loop diagrams (2.6) involving splittings

$$\varphi_{i_k}(p_k) \to V^a_\mu(q)\varphi_{i'_k}(p_k-q). \tag{3.1}$$

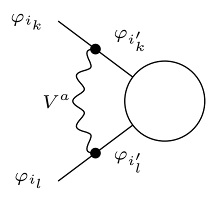
As mentioned in the previous section, we subtract soft contributions that give rise to singularities of the integrand $N(q)$ in the region $q^\mu \to 0$. These result from diagrams where the gauge boson V^a couples to another external leg φ_{i_l} in the eikonal approximation (the term in the third line of (3.2)). These soft contributions can be treated separately [7]. For the remaining SL collinear singularities we derive the factorization identities

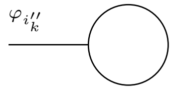
$$\delta^{\mathrm{coll}}\mathcal{M}^{\varphi_{i_k}}(p_k) =$$

$$= \sum_{V^a=A,Z,W^{\pm}}\sum_{\varphi_{i'_k}}\left\{\left[\vphantom{\sum}\right.\right.$$



$$\left.\vphantom{\sum}\right]_{\mathrm{trunc.}}$$

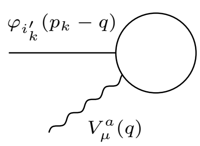
$$-\sum_{l\neq k}\sum_{\varphi_{i'_l}}\Bigg[$$  $$\Bigg]_{\text{eik.}}\Bigg\}_{\text{coll.}}$$

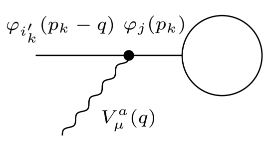
$$=\sum_{\varphi_{i''_k}}$$  $$\delta^{\text{coll}}_{\varphi_{i''_k}\varphi_{i_k}}\,, \qquad (3.2)$$

where the curly bracket, consisting of truncated (trunc.) diagrams and subtracted eikonal contributions (eik.), is evaluated in collinear approximation (coll.). The sum over V^a extends over W^+ and W^-, although in many cases only one of them contributes. The detailed proof of (3.2) depends on the spin of the external particles, which may be scalar bosons ($\varphi_i = \Phi_i$), transverse gauge bosons ($\varphi_i = V^a_{\mathrm{T}}$), or fermions ($\varphi_i = \Psi^{\kappa}_{j,\sigma}$). However, its basic structure can be sketched in a universal way and consists of two main steps:

(1) After insertion of the expressions for the explicit vertices and propagators, explicit subtraction of the eikonal contributions, and in the limit of collinear gauge-boson emission, the l.h.s. of (3.2) turns into[1]

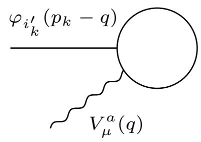
$$\delta^{\text{coll}}\mathcal{M}^{\varphi_{i_k}}(p_k) = \sum_{V^a=A,Z,W^\pm}\sum_{\varphi_{i'_k}}\mu^{4-D} \times\int\frac{\mathrm{d}^D q}{(2\pi)^D}\frac{-\mathrm{i}eI^{\bar{V}^a}_{\varphi_{i'_k}\varphi_{i_k}}}{(q^2-M^2_{V^a})[(p_k-q)^2-M^2_{\varphi_{i'_k}}]}$$

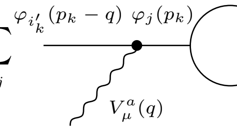
$$\times K_{\varphi_{i_k}}\lim_{q^\mu\to xp^\mu_k} q^\mu\Bigg\{$$ 

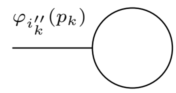
$$-\sum_{\varphi_j}$$  $$\Bigg\}, \qquad (3.3)$$

where $\mathrm{i}eI^{V^a}_{\varphi_{i'}\varphi_i}$ is the coupling corresponding to the $V^a\bar{\varphi}_{i'}\varphi_i$ vertex with all fields incoming. The I^{V^a} are the generators of $\mathrm{SU}(2)\times\mathrm{U}(1)$ transformations of the fields φ_{i_k} and are discussed in detail in Appendix B of [7]. The charge-conjugate fields are denoted by $\bar{\varphi}_{i_k}$. For scalar bosons and transverse gauge bosons $K_{\varphi_{i_k}} = 1$, while for fermions $K_{\varphi_{i_k}} = 2$. The first diagram appearing in (3.3) results from the first diagram of (3.2) by omitting the explicit vertex and propagators. The second diagram in (3.3) originates from the truncation of the self-energy and mixing-energy ($\varphi_i\varphi_j$) insertions in the first diagram of (3.2). Equation (3.3) is derived in Sects. 3.1–3.3.

(2) The contraction of the diagrams between the curly brackets on the r.h.s. of (3.3) with the gauge-boson momentum q^μ can be simplified using the Ward identities

$$\lim_{q^\mu\to xp^\mu_k} q^\mu\Bigg\{$$ 

$$-\sum_{\varphi_j}$$  $$\Bigg\} =$$

$$=\sum_{\varphi_{i''_k}}$$  $$eI^{V^a}_{\varphi_{i''_k}\varphi_{i'_k}}\,, \qquad (3.4)$$

which are fulfilled in the collinear approximation and valid up to mass-suppressed terms. These Ward identities are derived in Sect. 4 using the BRS invariance of the spontaneously broken $\mathrm{SU}(2)\times\mathrm{U}(1)$ Lagrangian.

Combining (3.4) with (3.3), we obtain (3.2) with the collinear factor

$$\begin{aligned}\delta^{\text{coll}}_{\varphi_{i''}\varphi_i} &= \sum_{V^a=A,Z,W^\pm}\sum_{\varphi_{i'}}\mu^{4-D} \\ &\quad\times\int\frac{\mathrm{d}^D q}{(2\pi)^D}\frac{-\mathrm{i}K_{\varphi_i}e^2 I^{V^a}_{\varphi_{i''}\varphi_{i'}}I^{\bar{V}^a}_{\varphi_{i'}\varphi_i}}{(q^2-M^2_{V^a})[(p-q)^2-M^2_{\varphi_{i'}}]} \\ &\overset{\text{LA}}{=} \frac{\alpha}{4\pi}K_{\varphi_i}\left[C^{\text{ew}}_{\varphi_{i''}\varphi_i}\log\frac{\mu^2}{M^2_W}+\delta_{\varphi_{i''}\varphi_i}Q^2_{\varphi_i}\log\frac{M^2_W}{M^2_{\varphi_i}}\right],\end{aligned} \qquad (3.5)$$

where the effective electroweak Casimir operator is defined by

$$\sum_{V^a=A,Z,W^\pm}\sum_{\varphi_{i'}} I^{V^a}_{\varphi_{i''}\varphi_{i'}}I^{\bar{V}^a}_{\varphi_{i'}\varphi_i} = C^{\text{ew}}_{\varphi_{i''}\varphi_i} \qquad (3.6)$$

and explicitly given in Appendix B of [7]. The integral is evaluated in Appendix A. For virtual massive gauge bosons $V^a = Z, W^\pm$, the scale of the logarithm is determined by M_W, for photons by the mass M_{φ_i} of the external particles.

In this section, we use the collinear Ward identities to derive (3.2) for external scalars, transverse gauge bosons,

[1] Here and in the following the $+\mathrm{i}\varepsilon$ prescription of the propagators is suppressed in the notation

and fermions. To this end, we introduce the following shorthand notation for matrix elements (2.1)

$$\mathcal{M}^{\varphi_{i_k}}(p_k) = v_{\varphi_{i_k}}(p_k) G^{\underline{\varphi_{i_k}}\,\underline{O}}(p_k), \tag{3.7}$$

i.e. we concentrate on a specific external leg φ_{i_k}, and only its momentum p_k and wave function $v_{\varphi_{i_k}}(p_k)$ are kept explicit. The wave function $v_{\varphi_{i_k}}(p_k)$ equals 1 for scalars and is given by the Dirac spinors for fermions and the polarization vectors for gauge bosons. It is contracted with the truncated Green function G (underlined field arguments correspond to truncated external legs; other conventions concerning Green functions are given in Appendix C). The operator

$$O(r) = \prod_{l \neq k} \varphi_{i_l}(p_l), \quad r = \sum_{l \neq k} p_l, \tag{3.8}$$

represents the remaining external legs. The external lines corresponding to the operator O are always assumed to be on-shell and contracted with the corresponding wave functions. These wave functions are always suppressed in the notation. Moreover, often also the operator O and the corresponding total momentum r are not written.

Note that in intermediate results, owing to gauge-boson emission $\varphi_{i_k} \to V^a \varphi_{i'_k}$, the matrix elements (3.7) are modified into expressions where the wave function $v_{\varphi_{i_k}}(p_k)$ with mass $p_k^2 = M^2_{\varphi_{i_k}}$ is contracted with a line $\varphi_{i'_k}$ carrying a different mass $M^2_{\varphi_{i'_k}}$. In the limit $s \gg M^2_{\varphi_{i_k}}, M^2_{\varphi_{i'_k}}$, the modified matrix elements are identified with matrix elements for $\varphi_{i'_k}$, since

$$v_{\varphi_{i_k}}(p_k) G^{\underline{\varphi_{i'_k}}\,\underline{O}}(p_k) = \mathcal{M}^{\varphi_{i'_k}}(p_k) + \mathcal{O}\left(\frac{M^2}{s}\mathcal{M}^{\varphi_{i'_k}}\right). \tag{3.9}$$

For the Green functions corresponding to the diagrams within the curly brackets in (3.3) we introduce the shorthand

$$G_\mu^{[\underline{V^a}\,\underline{\varphi_i}]\underline{O}}(q, p-q, r) = G_\mu^{\underline{V^a}\,\underline{\varphi_i}\,\underline{O}}(q, p-q, r) - \sum_{\varphi_j} G_\mu^{\underline{V^a}\,\underline{\varphi_i}\,\varphi_j}(q, p-q, -p) G^{\underline{\varphi_j}\,\underline{O}}(p, r). \tag{3.10}$$

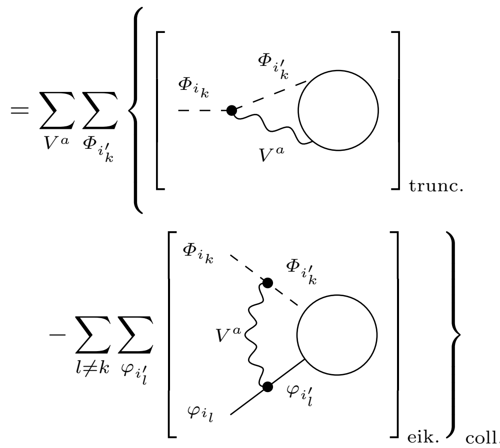
3.1 Factorization for scalars

We first consider the collinear enhancements generated by the virtual splittings

$$\Phi_{i_k}(p_k) \to V^a_\mu(q) \Phi_{i'_k}(p_k - q), \tag{3.11}$$

where an incoming on-shell Higgs boson or would-be Goldstone boson $\Phi_{i_k} = H, \chi, \phi^\pm$ emits a virtual collinear gauge boson $V^a = A, Z, W^\pm$. The corresponding amplitude is given by

$\delta^{\mathrm{coll}} \mathcal{M}^{\Phi_{i_k}}(p_k) =$



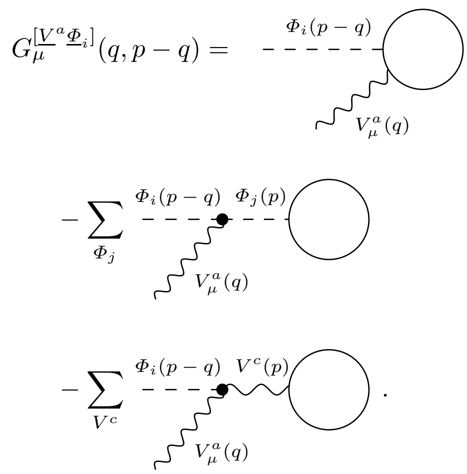
(3.12)

and reads

$$\delta^{\mathrm{coll}} \mathcal{M}^{\Phi_{i_k}}(p_k) = \sum_{V^a = A,Z,W^\pm} \sum_{\Phi_{i'_k} = H,\chi,\phi^\pm} \mu^{4-D} \times \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\mathrm{i} e I^{\bar{V}^a}_{\Phi_{i'_k} \Phi_{i_k}}}{(q^2 - M^2_{V^a})[(p_k - q)^2 - M^2_{\Phi_{i'_k}}]} \times \lim_{q^\mu \to x p_k^\mu} \left\{ (2p_k - q)^\mu G_\mu^{[\underline{V^a}\,\underline{\Phi_{i'_k}}]}(q, p_k - q) + 2p_k^\mu \sum_{l \neq k} \sum_{\varphi_{i'_l}} \frac{2e p_{l\mu} I^{V^a}_{\varphi_{i'_l} \varphi_{i_l}}}{[(p_l + q)^2 - M^2_{\varphi_{i'_l}}]} \times \mathcal{M}^{\Phi_{i'_k} \varphi_{i'_l}}(p_k, p_l) \right\}. \tag{3.13}$$

According to the definition (3.10), we have

$G_\mu^{[\underline{V^a}\,\underline{\Phi_i}]}(q, p-q) =$



(3.14)

Note that the subtracted contributions, when inserted in (3.12), correspond to external scalar self-energies ($\Phi\Phi$) and scalar–vector mixing-energies (ΦV).

We first concentrate on the expression between the curly brackets in (3.13), which has to be evaluated in the collinear limit $q^\mu \to x p_k^\mu$. Using

$$\lim_{q^\mu \to x p_k^\mu} \frac{2p_k p_l}{[(p_l+q)^2 - M^2_{\varphi_{i'_l}}]} = \frac{1}{x} + \mathcal{O}\left(\frac{M^2}{s}\right) \tag{3.15}$$

and $p_k^\mu \to q^\mu/x$, one finds[2]

$$\begin{aligned}\lim_{q^\mu \to x p_k^\mu} \{\ldots\} &= \lim_{q^\mu \to x p_k^\mu} \left\{ \left(\frac{2}{x} - 1\right) q^\mu G_\mu^{[V^a \Phi_{i'_k}]}(q, p_k - q) \right. \\ &\quad \left. + \frac{2e}{x} \sum_{l \neq k} \sum_{\varphi_{i'_l}} I^{V^a}_{\varphi_{i'_l}\varphi_{i_l}} \mathcal{M}^{\Phi_{i'_k}\varphi_{i'_l}}(p_k, p_l) \right\}. \end{aligned} \tag{3.16}$$

With the collinear Ward identity (4.29) for scalar bosons ($\varphi_i = \Phi_i$), this becomes

$$\begin{aligned}\lim_{q^\mu \to x p_k^\mu} \{\ldots\} &= -e \sum_{\Phi_{i''_k}} I^{V^a}_{\Phi_{i''_k}\Phi_{i'_k}} \mathcal{M}^{\Phi_{i''_k}}(p_k) \\ &\quad + \frac{2e}{x} \left\{ \sum_{\Phi_{i''_k}} I^{V^a}_{\Phi_{i''_k}\Phi_{i'_k}} \mathcal{M}^{\Phi_{i''_k}}(p_k) \right. \\ &\quad \left. + \sum_{l \neq k} \sum_{\varphi_{i'_l}} I^{V^a}_{\varphi_{i'_l}\varphi_{i_l}} \mathcal{M}^{\Phi_{i'_k}\varphi_{i'_l}}(p_k, p_l) \right\}. \end{aligned} \tag{3.17}$$

We now use global SU(2) × U(1) invariance, which leads to

$$\begin{aligned} \mathrm{i}e \sum_{k=1}^{n} \sum_{\varphi_{i'_k}} I^{V^a}_{\varphi_{i'_k}\varphi_{i_k}} \mathcal{M}^{\varphi_{i_1}\ldots\varphi_{i'_k}\ldots\varphi_{i_n}} \\ = \mathcal{O}\left(\frac{M^2}{s} \mathcal{M}^{\varphi_{i_1}\ldots\varphi_{i_k}\ldots\varphi_{i_n}}\right) \end{aligned} \tag{3.18}$$

for non-mass-suppressed matrix elements. With this, the part proportional to $1/x$ is mass-suppressed as expected, since the soft-photon contributions have been subtracted. Thus, (3.13) turns into

$$\begin{aligned}\delta^{\mathrm{coll}} \mathcal{M}^{\Phi_{i_k}}(p_k) &= \sum_{V^a, \Phi_{i'_k}, \Phi_{i''_k}} \mu^{4-D} \\ &\quad \times \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{-\mathrm{i}e^2 I^{V^a}_{\Phi_{i''_k}\Phi_{i'_k}} I^{\bar{V}^a}_{\Phi_{i'_k}\Phi_{i_k}}}{(q^2 - M^2_{V^a})[(p_k-q)^2 - M^2_{\Phi_{i'_k}}]} \mathcal{M}^{\Phi_{i''_k}}(p_k), \end{aligned} \tag{3.19}$$

and with (3.5) and $M_{\Phi_{i'_k}} \sim M_W$ we obtain the collinear factor

$$\delta^{\mathrm{coll}}_{\Phi_{i''}\Phi_i} \overset{\mathrm{LA}}{=} \frac{\alpha}{4\pi} \delta_{\Phi_{i''}\Phi_i} C^{\mathrm{ew}}_{\Phi} \log \frac{\mu^2}{M_W^2} \tag{3.20}$$

in LA. For external Higgs bosons, this has to be combined with the Higgs FRC [7] as in (2.4). The resulting collinear correction factor reads

$$\delta^{\mathrm{C}}_{HH} = \frac{\alpha}{4\pi} \left[2C^{\mathrm{ew}}_{\Phi} - \frac{3}{4 s_{\mathrm{W}}^2} \frac{m_t^2}{M_W^2} \right] \log \frac{s}{M_W^2}, \tag{3.21}$$

where s_{W} represents the sine of the weak mixing angle. The collinear correction factors for external longitudinal gauge bosons are obtained from (3.20) and from the corrections to the equivalence theorem [7], and read

$$\begin{aligned}\delta^{\mathrm{C}}_{V_{\mathrm{L}}^{b''} V_{\mathrm{L}}^{b}} &= \delta_{V^{b''}V^b} \frac{\alpha}{4\pi} \left\{ \left[2C^{\mathrm{ew}}_{\Phi} - \frac{3}{4 s_{\mathrm{W}}^2} \frac{m_t^2}{M_W^2} \right] \right. \\ &\quad \left. \times \log \frac{s}{M_W^2} + Q^2_{V^b} \log \frac{M_W^2}{\lambda^2} \right\}. \end{aligned} \tag{3.22}$$

As pointed out in [7], Higgs bosons and longitudinal gauge bosons receive the same collinear SL corrections. The difference between (3.21) and (3.22) consists only in an electromagnetic soft contribution, which is contained in the FRC for charged gauge bosons and depends on the infinitesimal photon mass λ. This suggests that the logarithmic corrections for longitudinal gauge bosons can be reproduced in the symmetric approach. In fact, the result (3.22) is equivalent to

$$\delta^{\mathrm{C}}_{V_{\mathrm{L}}^{b'} V_{\mathrm{L}}^{b}} \overset{\mathrm{LA}}{=} \left(\frac{1}{2} \delta Z_{\Phi_{b'}\Phi_b} + \delta^{\mathrm{coll}}_{\Phi_{b'}\Phi_b} \right) \Big|_{\mu^2 = s}, \tag{3.23}$$

if a FRC for on-shell would-be Goldstone bosons

$$\delta Z_{\Phi_{b'}\Phi_b} = - \left(\frac{\partial}{\partial p^2} \Sigma_{\Phi_{b'}\Phi_b}(p^2) \right) \Big|_{p^2 = M^2_{V^b}}, \tag{3.24}$$

is used where, however, the contributions from $\mathcal{L}_v$ have to be omitted. Note that (3.23) corresponds to the collinear factor for physical scalar bosons belonging to a Higgs doublet with vanishing vev. This result can be interpreted as follows: as far as logarithmic one-loop corrections are concerned, at high energies the longitudinal gauge bosons can be described by would-be Goldstone bosons as physical scalar bosons. This justifies the symmetric approach at the one-loop leading-logarithmic level.

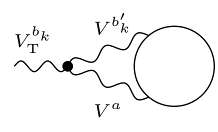
3.2 Factorization for transverse gauge bosons

Next, we consider the collinear enhancements generated by the virtual splittings

$$V_\nu^{b_k}(p_k) \to V_\mu^a(q) V_{\nu'}^{b'_k}(p_k - q), \tag{3.25}$$

where an incoming on-shell transverse gauge boson $V_{\mathrm{T}}^{b_k} = A_{\mathrm{T}}, Z_{\mathrm{T}}, W_{\mathrm{T}}^{\pm}$ emits a virtual collinear gauge boson $V^a = A, Z, W^{\pm}$. The corresponding amplitude is given by

$$\delta^{\mathrm{coll}} \mathcal{M}^{V_{\mathrm{T}}^{b_k}}(p_k) =$$

$$= \frac{1}{2} \sum_{V^a, V^{b'_k}} \Bigg\{ \Bigg[$$  $$\Bigg]_{\mathrm{trunc.}}$$

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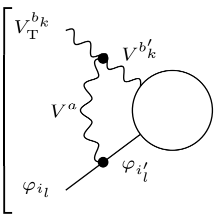
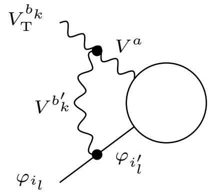
[2] Since the soft-photon contributions are subtracted, we do not need a regularization of $1/x$ for $x \to 0$

$$-\sum_{l\neq k}\sum_{\varphi_{i'_l}}\Bigg[$$



$$+$$



$$\Bigg]_{\text{eik.}}\Bigg\}_{\text{coll.}}. \tag{3.26}$$

The r.h.s. of (3.26) is manifestly symmetric with respect to an interchange of the gauge bosons V^a and $V^{b'_k}$ resulting from the splitting (3.25). In particular, the subtracted eikonal contributions are decomposed into terms originating from soft V^a bosons ($q^\mu \to 0$) as well as from soft $V^{b'_k}$ bosons ($q^\mu \to p_k^\mu$). The symmetry factor 1/2 compensates double counting in the sum over $V^a, V^{b'_k} = A, Z, W^\pm$. The resulting amplitude is

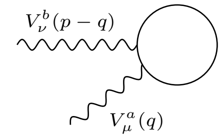
$$\begin{aligned}
\delta^{\mathrm{coll}}\mathcal{M}^{V_{\mathrm{T}}^{b_k}}(p_k) &= \frac{1}{2}\sum_{V^a,V^{b'_k}}\mu^{4-D}\\
&\times\int\frac{\mathrm{d}^D q}{(2\pi)^D}\frac{\mathrm{i}eI^{\bar{V}^a}_{V^{b'_k}V^{b_k}}}{(q^2-M^2_{V^a})[(p_k-q)^2-M^2_{V^{b'_k}}]}\\
&\times\lim_{q^\mu\to xp_k^\mu}\varepsilon_{\mathrm{T}\nu}(p_k)\Bigg\{F^{\mu\nu\nu'}(q,p_k-q)\\
&\times G^{[\underline{V}^a\underline{V}^{b'_k}]}_{\mu\nu'}(q,p_k-q)\\
&+\sum_{l\neq k}\sum_{\varphi_{i'_l}}\Bigg[F^{\mu\nu\nu'}(0,p_k)\frac{2ep_{l\mu}I^{V^a}_{\varphi_{i'_l}\varphi_{i_l}}}{[(p_l+q)^2-M^2_{\varphi_{i'_l}}]}\\
&\times G^{\underline{V}^{b'_k}\underline{\varphi}_{i'_l}}_{\nu'}(p_k,p_l)\\
&+F^{\mu\nu\nu'}(p_k,0)\frac{2ep_{l\nu'}I^{V^{b'_k}}_{\varphi_{i'_l}\varphi_{i_l}}}{[(p_l+p_k-q)^2-M^2_{\varphi_{i'_l}}]}\\
&\times G^{\underline{V}^a\underline{\varphi}_{i'_l}}_{\mu}(p_k,p_l)\Bigg]v_{\varphi_{i_l}(p_l)}\Bigg\},
\end{aligned} \tag{3.27}$$

where

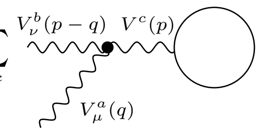
$$\begin{aligned}
F^{\mu\nu\nu'}(q,p_k-q) = \Big[&g^{\nu\nu'}(2p_k-q)^\mu + g^{\nu'\mu}(2q-p_k)^\nu\\
&-g^{\mu\nu}(p_k+q)^{\nu'}\Big]
\end{aligned} \tag{3.28}$$

is the vertex function associated to the splitting (3.25). According to the definition (3.10),

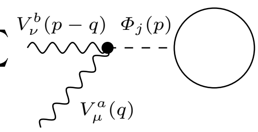
$$G^{[\underline{V}^a\underline{V}^b]}_{\mu\nu}(q,p-q) =$$



$$-\sum_{V^c}$$



$$-\sum_{\Phi_j}$$



$$. \tag{3.29}$$

We first concentrate on the contraction of the vertex (3.28) with the transverse polarization vector $\varepsilon^\nu_{\mathrm{T}}(p_k)$. Owing to $p^\nu_k\varepsilon_{\mathrm{T}\nu}(p_k) = 0$, the second term on the r.h.s. of (3.28) vanishes in the collinear limit $q^\mu \to xp^\mu_k$, and

$$\begin{aligned}
&\lim_{q^\mu\to xp_k^\mu}\varepsilon_{\mathrm{T}\nu}(p_k)F^{\mu\nu\nu'}(q,p_k-q)\\
&=\left(\frac{2}{x}-1\right)\varepsilon^{\nu'}_{\mathrm{T}}(p_k)q^\mu-\left(\frac{2}{1-x}-1\right)\varepsilon^\mu_{\mathrm{T}}(p_k)(p_k-q)^{\nu'}.
\end{aligned} \tag{3.30}$$

In the fractions $2/x$ and $2/(1-x)$ we have isolated the terms leading to IR enhancements at $x \to 0$ and $x \to 1$, respectively. These must be cancelled by the subtracted eikonal contributions, i.e. the terms in the last four lines in (3.27). In these contributions some terms are mass-suppressed or vanishing owing to the following identities for the massive or massless on-shell gauge bosons $V^{b'_k}$ and V^a, respectively,

$$\begin{aligned}
\varepsilon^\mu_{\mathrm{T}}(p_k)p_{l\mu}p^{\nu'}_k G^{\underline{V}^{b'_k}\underline{\varphi}_{i'_l}}_{\nu'}(p_k,p_l) &\sim M_{V^{b'_k}}\mathcal{M}^{V_{\mathrm{L}}^{b'_k}\varphi_{i'_l}}(p_k,p_l),\\
\varepsilon^{\nu'}_{\mathrm{T}}(p_k)p_{l\nu'}p^\mu_k G^{\underline{V}^a\underline{\varphi}_{i'_l}}_{\mu}(p_k,p_l) &\sim M_{V^a}\mathcal{M}^{V_{\mathrm{L}}^a\varphi_{i'_l}}(p_k,p_l).
\end{aligned} \tag{3.31}$$

Thus, the relevant terms are obtained by the substitutions

$$\begin{aligned}
\varepsilon_{\mathrm{T}\nu}(p_k)F^{\mu\nu\nu'}(0,p_k) &\to 2\varepsilon^{\nu'}_{\mathrm{T}}(p_k)p^\mu_k,\\
\varepsilon_{\mathrm{T}\nu}(p_k)F^{\mu\nu\nu'}(p_k,0) &\to -2\varepsilon^\mu_{\mathrm{T}}(p_k)p^{\nu'}_k
\end{aligned} \tag{3.32}$$

in (3.27). With (3.30) and (3.32), the expression between the curly brackets on the r.h.s. of (3.27) gives

$$\begin{aligned}
&\lim_{q^\mu\to xp_k^\mu}\{\dots\}\\
&=\lim_{q^\mu\to xp_k^\mu}\Bigg\{\Bigg[\left(\frac{2}{x}-1\right)\varepsilon^{\nu'}_{\mathrm{T}}(p_k)q^\mu-\left(\frac{2}{1-x}-1\right)\\
&\times\varepsilon^\mu_{\mathrm{T}}(p_k)(p-q)^{\nu'}\Bigg]G^{[\underline{V}^a\underline{V}^{b'_k}]}_{\mu\nu'}(q,p_k-q)
\end{aligned}$$

$$+\sum_{l\neq k}\sum_{\varphi_{i'_l}}\left[\frac{2e}{x}I^{V^a}_{\varphi_{i'_l}\varphi_{i_l}}\mathcal{M}^{V_{\mathrm{T}}^{b'_k}\varphi_{i'_l}}(p_k,p_l)\right.$$
$$\left.\left.-\frac{2e}{1-x}I^{V^{b'_k}}_{\varphi_{i'_l}\varphi_{i_l}}\mathcal{M}^{V_{\mathrm{T}}^{a}\varphi_{i'_l}}(p_k,p_l)\right]\right\}. \qquad (3.33)$$

Using the collinear Ward identity (4.29) for gauge bosons ($\varphi_i = V^b_\nu$) and the equivalent identity

$$\lim_{q^\mu\to xp^\mu}\varepsilon^\mu_{\mathrm{T}}(p)(p-q)^\nu G^{[\underline{V}^a\underline{V}^b]\underline{O}}_{\mu\nu}(q,p-q,r)$$
$$= e\sum_{V^{b'}}\mathcal{M}^{V_{\mathrm{T}}^{b'}O}(p,r)I^{V^b}_{V^{b'}V^a}, \qquad (3.34)$$

(3.33) simplifies into

$$\lim_{q^\mu\to xp^\mu_k}\{\ldots\} = -e\sum_{V^{b''_k}}\left[I^{V^a}_{V^{b''_k}V^{b'_k}} - I^{V^{b'_k}}_{V^{b''_k}V^a}\right]\mathcal{M}^{V_{\mathrm{T}}^{b''_k}}(p_k)$$
$$+\frac{2e}{x}\left(\sum_{V^{b''_k}}I^{V^a}_{V^{b''_k}V^{b'_k}}\mathcal{M}^{V_{\mathrm{T}}^{b''_k}}(p_k)\right.$$
$$\left.+\sum_{l\neq k}\sum_{\varphi_{i'_l}}I^{V^a}_{\varphi_{i'_l}\varphi_{i_l}}\mathcal{M}^{V_{\mathrm{T}}^{b'_k}\varphi_{i'_l}}(p_k,p_l)\right)$$
$$-\frac{2e}{1-x}\left(\sum_{V^{b''_k}}I^{V^{b'_k}}_{V^{b''_k}V^a}\mathcal{M}^{V_{\mathrm{T}}^{b''_k}}(p_k)\right.$$
$$\left.+\sum_{l\neq k}\sum_{\varphi_{i'_l}}I^{V^{b'_k}}_{\varphi_{i'_l}\varphi_{i_l}}\mathcal{M}^{V_{\mathrm{T}}^{a}\varphi_{i'_l}}(p_k,p_l)\right). \qquad (3.35)$$

Again, the soft terms proportional to $1/x$ and $1/(1-x)$ are mass-suppressed owing to global SU(2) × U(1) invariance (3.18), so that only the first term in (3.35) remains. Inserting this into (3.27) with $I^{V^b}_{V^cV^a} = -I^{V^a}_{V^cV^b}$, we find

$$\delta^{\mathrm{coll}}\mathcal{M}^{V_{\mathrm{T}}^{b_k}}(p_k) = \sum_{V^a,V^{b'_k},V^{b''_k}}\mu^{4-D} \qquad (3.36)$$
$$\times\int\frac{\mathrm{d}^Dq}{(2\pi)^D}\frac{-\mathrm{i}e^2 I^{V^a}_{V^{b''_k}V^{b'_k}}I^{\bar{V}^a}_{V^{b'_k}V^{b_k}}}{(q^2-M^2_{V^a})[(p_k-q)^2-M^2_{V^{b'_k}}]}\mathcal{M}^{V_{\mathrm{T}}^{b''_k}}(p_k).$$

With (3.5) we obtain the collinear factor

$$\delta^{\mathrm{coll}}_{V_{\mathrm{T}}^{b''_k}V_{\mathrm{T}}^{b_k}} \overset{\mathrm{LA}}{=} \frac{\alpha}{4\pi}C^{\mathrm{ew}}_{V^{b''_k}V^{b_k}}\log\frac{\mu^2}{M_W^2} \qquad (3.37)$$

in LA. The complete SL collinear (and soft) correction factors for transverse gauge bosons are obtained by including the corresponding FRC's given in [7] and read

$$\delta^{\mathrm{C}}_{V_{\mathrm{T}}^aV_{\mathrm{T}}^b} = \frac{\alpha}{4\pi}\left\{\frac{1}{2}\left[b^{\mathrm{ew}}_{V^aV^b} + E_{V^aV^b}b^{\mathrm{ew}}_{AZ}\right]\log\frac{s}{M_W^2}\right.$$
$$\left.+\delta_{V^aV^b}Q^2_{V^a}\log\frac{M_W^2}{\lambda^2}\right\}$$
$$-\frac{1}{2}\delta_{V^aA}\delta_{V^bA}\Delta\alpha(M_W^2). \qquad (3.38)$$

The s-dependent part is determined by the one-loop coefficients $b^{\mathrm{ew}}_{V^aV^b}$ of the electroweak β-function (see [7]). Furthermore, $E_{V^aV^b}$ is an antisymmetric matrix with non-vanishing components $E_{AZ} = -E_{ZA} = 1$. The remaining terms represent a soft contribution proportional to the charge of the gauge boson and a pure electromagnetic contribution originating from light-fermion loops that can be related to the running of the electromagnetic coupling from zero to the scale M_W (defined explicitly in [7]).

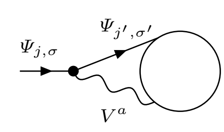
3.3 Factorization for fermions

We finally consider the collinear enhancements generated by the virtual splittings

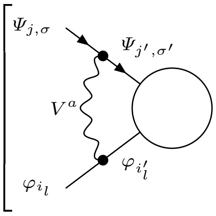
$$f^\kappa_{j,\sigma}(p_k) \to V^a_\mu(q)f^\kappa_{j',\sigma'}(p_k-q), \qquad (3.39)$$

where a virtual collinear gauge boson $V^a = A, Z, W^\pm$ is emitted by an incoming on-shell fermion $f^\kappa_{j,\sigma}$, i.e. a quark or lepton $f = Q, L$, with chirality $\kappa = \mathrm{L}, \mathrm{R}$, isospin index $\sigma = \pm$, and generation index $j = 1, 2, 3$. The collinear singularity is contained in

$$\delta^{\mathrm{coll}}\mathcal{M}^{f^\kappa_{j,\sigma}}(p_k)$$
$$= \sum_{V^a}\sum_{j'\sigma'}\left\{\left[\right.\right.$$



$$\left.\right]_{\mathrm{trunc.}}$$
$$-\sum_{l\neq k}\sum_{\varphi_{i'_l}}\left[\right.$$



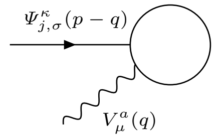
$$\left.\left.\right]_{\mathrm{eik.}}\right\}_{\mathrm{coll.}}. \qquad (3.40)$$

The corresponding amplitude reads

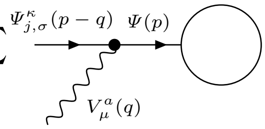
$$\delta^{\mathrm{coll}}\mathcal{M}^{f^\kappa_{j,\sigma}}(p_k) = \sum_{V^a=A,Z,W^\pm}\sum_{j',\sigma'}\mu^{4-D}$$
$$\times\int\frac{\mathrm{d}^Dq}{(2\pi)^D}\frac{\mathrm{i}eI^{\bar{V}^a}_{\sigma'\sigma}U^{\bar{V}^a}_{j'j}}{(q^2-M^2_{V^a})[(p_k-q)^2-m^2_{f_{j',\sigma'}}]}$$
$$\times\lim_{q^\mu\to xp^\mu_k}\left\{\left[G^{[\underline{V}^a\underline{\Psi}^\kappa_{j',\sigma'}]}_\mu(q,p_k-q)(\not{p}_k-\not{q})\right.\right.$$
$$+\sum_{l\neq k}\sum_{\varphi_{i'_l}}\frac{2ep_{l\mu}I^{V^a}_{\varphi_{i'_l}\varphi_{i_l}}}{[(p_l+q)^2-M^2_{\varphi_{i'_l}}]}$$
$$\left.\left.\times G^{\underline{\Psi}^\kappa_{j',\sigma'}\underline{\varphi}_{i'_l}}(p_k,p_l)v_{\varphi_{i_l}}(p_l)\not{p}_k\right]\gamma^\mu u(p_k)\right\}, \qquad (3.41)$$

where the fermion-mass terms in the numerator have been neglected, and the unitary mixing matrix U^{V^a} is defined in (B.8). According to the definition (3.10) the Green function $G_\mu^{[\underline{V}^a \underline{\Psi}^\kappa_{j,\sigma}]}$ is diagrammatically given by

$$G_\mu^{[\underline{V}^a \underline{\Psi}^\kappa_{j,\sigma}]}(q, p-q) =$$



$$-\sum_{\Psi}$$



$$. \qquad (3.42)$$

In the collinear limit, the expression between the curly brackets in (3.41) can be simplified using (3.15),

$$\lim_{q^\mu \to x p_k^\mu} (\not{p}_k - \not{q})\gamma^\mu u(p_k) = \left(\frac{2}{x} - 2\right) q^\mu u(p_k) + \mathcal{O}(m_{j,\sigma}) u(p_k), \quad (3.43)$$

and the collinear Ward identity (4.31). One obtains

$$\begin{aligned}
\lim_{q^\mu \to x p_k^\mu} \{\dots\}
&= \lim_{q^\mu \to x p_k^\mu} \left(\frac{2}{x} - 2\right) q^\mu G_\mu^{[\underline{V}^a \underline{\Psi}^\kappa_{j',\sigma'}]}(q, p_k - q) u(p_k) \\
&\quad + \frac{2e}{x} \sum_{l \neq k} \sum_{\varphi_{i'_l}} I^{V^a}_{\varphi_{i'_l} \varphi_{i_l}} \mathcal{M}^{f^\kappa_{j',\sigma'} \varphi_{i'_l}}(p_k, p_l) \\
&= -2e \sum_{j'',\sigma''} I^{V^a}_{\sigma''\sigma'} U^{V^a}_{j''j'} \mathcal{M}^{f^\kappa_{j'',\sigma''}}(p_k) \\
&\quad + \frac{2e}{x} \left\{ \sum_{j'',\sigma''} I^{V^a}_{\sigma''\sigma'} U^{V^a}_{j''j'} \mathcal{M}^{f^\kappa_{j'',\sigma''}}(p_k) \right. \\
&\quad \left. + \sum_{l \neq k} \sum_{\varphi_{i'_l}} I^{V^a}_{\varphi_{i'_l} \varphi_{i_l}} \mathcal{M}^{f^\kappa_{j',\sigma'} \varphi_{i'_l}}(p_k, p_l) \right\}. \qquad (3.44)
\end{aligned}$$

Again, the soft-photon contributions proportional to $1/x$ are mass-suppressed owing to global gauge invariance (3.18). Thus, only the part originating from the $\not{q}$ term in (3.41) contributes, and we find

$$\begin{aligned}
\delta^{\mathrm{coll}} \mathcal{M}^{f^\kappa_{j,\sigma}}(p_k) &= \sum_{V^a, j', j'', \sigma', \sigma''} \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \qquad (3.45) \\
&\times \frac{-2\mathrm{i}e^2 I^{V^a}_{\sigma''\sigma'} U^{V^a}_{j''j'} I^{\bar{V}^a}_{\sigma'\sigma} U^{\bar{V}^a}_{j'j}}{(q^2 - M^2_{V^a})[(p_k - q)^2 - m^2_{f_{j',\sigma'}}]} \mathcal{M}^{f^\kappa_{j'',\sigma''}}(p_k).
\end{aligned}$$

Using (3.5), and the unitarity of the mixing matrix, $\sum_{j'} U^{V^a}_{j''j'} U^{\bar{V}^a}_{j'j} = \delta_{j''j}$, the mixing matrix drops out, and we obtain the collinear factor in LA,

$$\begin{aligned}
\delta^{\mathrm{coll}}_{f^\kappa_{j'',\sigma''} f^\kappa_{j,\sigma}} &\overset{\mathrm{LA}}{=} \delta_{j''j} \delta_{\sigma''\sigma} \frac{\alpha}{2\pi} \\
&\times \left\{ C^{\mathrm{ew}}_{f^\kappa_\sigma} \log \frac{\mu^2}{M_W^2} + Q^2_{f_{j,\sigma}} \log \frac{M_W^2}{m^2_{f_{j,\sigma}}} \right\}. \qquad (3.46)
\end{aligned}$$

Adding the FRC for fermions [7], we obtain the SL collinear (and soft) corrections

$$\begin{aligned}
&\delta^{\mathrm{C}}_{f^\kappa_{j'',\sigma''} f^\kappa_{j,\sigma}} \\
&= \delta_{jj''} \delta_{\sigma\sigma''} \frac{\alpha}{4\pi} \left\{ \left[\frac{3}{2} C^{\mathrm{ew}}_{f^\kappa_\sigma} - \frac{1}{8 s_{\mathrm{W}}^2} \left((1 + \delta_{\kappa\mathrm{R}}) \frac{m^2_{f_{j,\sigma}}}{M_W^2} \right. \right. \right. \\
&\quad \left. \left. + \delta_{\kappa\mathrm{L}} \frac{m^2_{f_{j,-\sigma}}}{M_W^2} \right) \right] \log \frac{s}{M_W^2} \\
&\quad \left. + Q^2_{f_{j,\sigma}} \left[\frac{1}{2} \log \frac{M_W^2}{m^2_{f_{j,\sigma}}} + \log \frac{M_W^2}{\lambda^2} \right] \right\}. \qquad (3.47)
\end{aligned}$$

The Yukawa contributions are large only for external heavy quarks $f^\kappa_{j,\sigma} = t^{\mathrm{R}}$, t^{L}, and b^{L}. In contrast to the m_t^2 corrections to the ρ parameter, which are only related to the (virtual) left-handed (t, b) doublet, logarithmic Yukawa contributions appear also for (external) right-handed top quarks.

4 Collinear Ward identities

As we have already stressed in Sect. 3, the proof of the factorization identities (3.2) is based on the collinear Ward identities (3.4). In the compact notation introduced in (3.7) and (3.10), these Ward identities read

$$\begin{aligned}
&\lim_{q^\mu \to x p^\mu} q^\mu v_\varphi(p) G_\mu^{[\underline{V}^a \underline{\varphi}_i] \underline{O}}(q, p-q, r) \\
&= e \sum_{\varphi_{i'}} \mathcal{M}^{\varphi_{i'} O}(p, r) I^{V^a}_{\varphi'_i \varphi_i}, \qquad (4.1)
\end{aligned}$$

A detailed derivation of these identities is presented in Sect. 4.1, for external scalars ($\varphi_i = \Phi_i$) and gauge bosons ($\varphi_i = V^a$) and in Sect. 4.2 for fermions ($\varphi_i = \Psi^\kappa_{j,\sigma}$). Here we discuss the most important features and restrictions concerning the Ward identities (4.1):

(1) They are restricted to LO matrix elements. We stress that all equations used in this section are only valid in LO.
(2) They are realized in the high-energy limit (2.2), and in the limit of collinear gauge boson momenta q and quasi-on-shell external momenta p, i.e. in the limit where $0 < p^2, (p-q)^2 \ll s$. All these limits have to be taken simultaneously. The wave function $v_\varphi(p)$ corresponds to a particle with mass $(p^2)^{1/2}$.
(3) They are valid only up to mass-suppressed terms, to be precise terms of the order $M/s^{1/2}$ (for fermions) or M^2/s (for bosons) with respect to the leading terms appearing in (4.1), where $M^2 \sim \max(p^2, M^2_{\varphi_i}, M^2_{V^a})$. Furthermore, they apply only to matrix elements that are not mass-suppressed. In other words, they apply to those matrix elements that arise from $\mathcal{L}_{\mathrm{symm}}$ in LO.
(4) Their derivation is based on the BRS invariance of a spontaneously broken gauge theory (see Appendix B). In particular, we used only the generic form of the

BRS transformations of the fields, the form of the gauge-fixing term in an arbitrary 't Hooft gauge, (B.10), and the corresponding form of the tree-level propagators. Therefore, the result is valid for a general spontaneously broken gauge theory, in an arbitrary 't Hooft gauge.

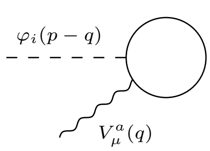
It is important to observe that the identities (4.1) do not reflect the presence of the non-vanishing vev of the Higgs doublet. In fact, they are identical to the identities obtained within a symmetric gauge theory with massless gauge bosons. However, spontaneous symmetry breaking plays a non-trivial role in ensuring the validity of (4.1). It guarantees the cancellation of mixing terms between gauge bosons and would-be Goldstone bosons. In particular, we stress the following: *extra contributions originating from $\mathcal{L}_v$ cannot be excluded a priori in (4.1).* In fact, the corresponding mass-suppressed couplings can in principle give extra leading contributions if they are enhanced by propagators with small invariants. We show that no such extra terms are left in the final result. Such terms appear, however, in the derivation of the Ward identity for external would-be Goldstone bosons ($\varphi_i = \Phi_i$) as "extra contributions" involving gauge bosons ($\varphi_{i'} = V^a$), and in the derivation of the Ward identity for external gauge bosons ($\varphi_i = V^a$) as "extra contributions" involving would-be Goldstone bosons ($\varphi_{i'} = \Phi_i$) [see (4.13)]. Their cancellation is ensured by Ward identities (4.18) relating the electroweak vertex functions that involve explicit factors with mass dimension. *In other words, the validity of (4.1) within a spontaneously broken gauge theory is a non-trivial consequence of the symmetry of the full theory.*

In the following, the collinear Ward identities are derived for matrix elements involving the physical fields of the electroweak theory.

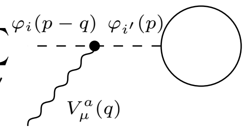
4.1 Scalar bosons and transverse gauge bosons

The Ward identities for external scalar bosons $\Phi_i = H, \chi, \phi^\pm$ and transverse gauge bosons $V^b = A, Z, W^\pm$ are of the same form. Here we derive a generic Ward identity for external bosonic fields φ_i valid for $\varphi_i = \Phi_i$ as well as $\varphi_i = V^b_\mu$. In both cases mixing between would-be Goldstone bosons and gauge bosons has to be taken into account[3]. Therefore, we use the symbol $\tilde{\varphi}$ to denote the mixing partner of φ, i.e. we have $(\varphi, \tilde{\varphi}) = (\Phi, V)$ or $(\varphi, \tilde{\varphi}) = (V, \Phi)$. The resulting Ward identities read

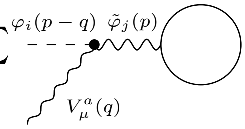
$\lim_{q^\mu \to x p^\mu} q^\mu \times \Bigg\{$



$- \sum_{\varphi_{i'}}$

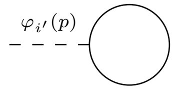


$- \sum_{\tilde{\varphi}_j}$



$\Bigg\} =$

$= e \sum_{\varphi_{i'}} I^{V^a}_{\varphi_{i'}\varphi_i}$



$+ \mathcal{O}\left(M^2 E^{d-2}\right),$ (4.2)

where $M^2 \sim \max(p^2, M^2_{\varphi_i}, M^2_{V^a})$, and d is the mass dimension of the matrix element $\mathcal{M}^{\varphi_i}$. The diagrammatic representation corresponds to external scalars ($\varphi = \Phi$). For the proof of (4.2), we start from the BRS invariance (cf. Appendix B) of the Green function $\langle \bar{u}^a(x) \varphi_i^+(y) O(z) \rangle$:

$$\begin{aligned} &\langle [s\bar{u}^a(x)] \varphi_i^+(y) O(z) \rangle - \langle \bar{u}^a(x) [s\varphi_i^+(y)] O(z) \rangle \\ &\quad = \langle \bar{u}^a(x) \varphi_i^+(y) [sO(z)] \rangle. \end{aligned} \tag{4.3}$$

With the BRS variations (B.15) and (B.13) this yields

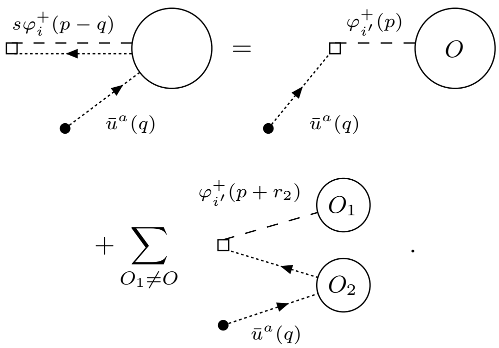
$$\begin{aligned} &\frac{1}{\xi_a} \partial^\mu_x \langle \bar{V}^a_\mu(x) \varphi_i^+(y) O(z) \rangle \\ &\qquad - \mathrm{i}ev \sum_{\Phi_j = H,\chi,\phi^\pm} I^{V^a}_{H\Phi_j} \langle \Phi_j(x) \varphi_i^+(y) O(z) \rangle \\ &\qquad + \sum_{V^b = A,Z,W^\pm} \Bigg[X^{V^b}_{\varphi_i^+} \langle \bar{u}^a(x) u^b(y) O(z) \rangle \\ &\qquad - \mathrm{i}e \sum_{\varphi_{i'}} \langle \bar{u}^a(x) u^b(y) \varphi_{i'}^+(y) O(z) \rangle I^{V^b}_{\varphi_{i'}\varphi_i} \Bigg] \\ &\quad = - \langle \bar{u}^a(x) \varphi_i^+(y) [sO(z)] \rangle. \end{aligned} \tag{4.4}$$

Fourier transformation of the variables (x, y, z) to the incoming momenta $(q, p-q, r)$ $(\partial^\mu_x \to \mathrm{i}q^\mu)$ gives

$$\begin{aligned} &\frac{\mathrm{i}}{\xi_a} q^\mu \langle \bar{V}^a_\mu(q) \varphi_i^+(p-q) O(r) \rangle \\ &\qquad - \mathrm{i}ev \sum_{\Phi_j} I^{V^a}_{H\Phi_j} \langle \Phi_j(q) \varphi_i^+(p-q) O(r) \rangle \\ &\qquad + \sum_{V^b} X^{V^b}_{\varphi_i^+} \langle \bar{u}^a(q) u^b(p-q) O(r) \rangle - \mathrm{i}e \sum_{V^b, \varphi_{i'}} I^{V^b}_{\varphi_{i'}\varphi_i} \\ &\qquad \times \int \frac{\mathrm{d}^D l}{(2\pi)^D} \langle \bar{u}^a(q) u^b(l) \varphi_{i'}^+(p-q-l) O(r) \rangle \\ &\quad = - \langle \bar{u}^a(q) \varphi_i^+(p-q) [sO(r)] \rangle. \end{aligned} \tag{4.5}$$

[3] For external Higgs bosons or photons all mixing terms vanish

From now on, the r.h.s. is omitted, since the BRS variation of on-shell physical fields does not contribute to physical matrix elements. This can be verified by truncation of the physical external legs $O(r)$ and contraction with the wave functions. A further simplification concerns the last term on the l.h.s. of (4.5). This originates from the BRS variation $s\varphi_i^+(y)$ of the external scalar or vector field and contains an external "BRS vertex" connecting the fields $u^b(y)\varphi_{i'}^+(y)$, which we represent by a small box in (4.6). When we restrict the relation to LO connected Green functions, this term simplifies into those tree diagrams where the external ghost line is not connected to the scalar leg of the BRS vertex by internal vertices,



(4.6)

We will see in the following that the relevant contributions result only from the first diagram on the r.h.s. of (4.6), where the ghosts are joined by a propagator and all on-shell legs $O(r)$ are connected to the leg $\varphi_{i'}^+$, which receives momentum $p=-r$. In the remaining diagrams, the on-shell legs are distributed into two subsets $O(r) = O_1(r_1)O_2(r_2)$ with momenta $r_1+r_2=r$. One subset O_1 interacts with the leg $\varphi_{i'}^+$, which receives momentum $p+r_2=-r_1$. The other subset O_2 interacts with the ghost line. Therefore, in LO the last term on the l.h.s. of (4.5) yields

$$\begin{aligned}
&-\mathrm{i}e\sum_{V^b,\varphi_{i'}} I^{V^b}_{\varphi_{i'}\varphi_i}\int\frac{\mathrm{d}^D l}{(2\pi)^D}\langle\bar{u}^a(q)u^b(l)\varphi_{i'}^+(p-q-l)O(r)\rangle\\
&\quad=-\mathrm{i}e\sum_{\varphi_{i'}}\langle\bar{u}^a(q)u^a(-q)\rangle\langle\varphi_{i'}^+(p)O(r)\rangle I^{V^a}_{\varphi_{i'}\varphi_i}\\
&\qquad-\mathrm{i}e\sum_{V^b,\varphi_{i'}}\sum_{O_1\neq O}\langle\bar{u}^a(q)u^b(-q-r_2)O_2(r_2)\rangle\\
&\qquad\times\langle\varphi_{i'}^+(p+r_2)O_1(r_1)\rangle I^{V^b}_{\varphi_{i'}\varphi_i},
\end{aligned}\tag{4.7}$$

and if we split off the momentum-conservation δ-functions, (4.5) becomes

$$\begin{aligned}
&\frac{\mathrm{i}}{\xi_a}q^\mu G^{\bar{V}^a\varphi_i^+\underline{O}}_\mu(q,p-q,r)\\
&\quad-\mathrm{i}ev\sum_{\Phi_j}I^{V^a}_{H\Phi_j}G^{\Phi_j\varphi_i^+\underline{O}}(q,p-q,r)\\
&\quad+\sum_{V^b}X^{V^b}_{\varphi_i^+}G^{\bar{u}^a u^b\underline{O}}(q,p-q,r)\\
&\quad-\mathrm{i}e\sum_{\varphi_{i'}}G^{\bar{u}^a u^a}(q)G^{\varphi_{i'}^+\underline{O}}(p,r)I^{V^a}_{\varphi_{i'}\varphi_i}\\
&\quad=\mathrm{i}e\sum_{V^b,\varphi_{i'}}\sum_{O_1\neq O}G^{\varphi_{i'}^+\underline{O}_1}(p+r_2,r_1)I^{V^b}_{\varphi_{i'}\varphi_i}\\
&\qquad\times G^{\bar{u}^a u^b\underline{O}_2}(q,-q-r_2,r_2).
\end{aligned}\tag{4.8}$$

Recall that we are interested in the on-shell and "massless" limit $p^2 \ll s$ of the above equation. Therefore, we have to take special care of all terms that are enhanced in this limit, like internal propagators carrying momentum p. Since internal lines with small invariants do not occur on the r.h.s. of (4.8), we now concentrate on the l.h.s. Using (3.10), the first term can be written as

$$\begin{aligned}
G^{\bar{V}^a\varphi_i^+\underline{O}}_\mu(q,p-q,r) &= G^{[\bar{V}^a\varphi_i^+]\underline{O}}_\mu(q,p-q,r)\\
&\quad+\sum_{\varphi_{i'}}G^{\bar{V}^a\varphi_i^+\varphi_{i'}}_\mu(q,p-q,-p)G^{\underline{\varphi}_{i'}\underline{O}}(p,r)\\
&\quad+\sum_{\tilde{\varphi}_j}G^{\bar{V}^a\varphi_i^+\tilde{\varphi}_j}_\mu(q,p-q,-p)G^{\underline{\tilde{\varphi}}_j\underline{O}}(p,r),
\end{aligned}\tag{4.9}$$

where for scalar φ_i^+ the sums run over scalar $\varphi_{i'}$ and vector $\tilde{\varphi}_j$ and vice versa if φ_i^+ is a vector. In this way the enhanced internal propagators with momentum p are isolated in the terms $G^{\bar{V}^a\varphi_i^+\varphi_{i'}}_\mu(q,p-q,-p)$ and $G^{\bar{V}^a\varphi_i^+\tilde{\varphi}_j}_\mu(q,p-q,-p)$, whereas the subtracted Green functions $G^{[\bar{V}^a\varphi_i^+]\underline{O}}_\mu$ contain no enhancement by definition. A similar decomposition is used for the second and third term on the l.h.s. of (4.8), whereas the enhanced propagator contained in the last term is isolated by writing

$$G^{\varphi_{i'}^+\underline{O}}(p,r)=G^{\varphi_{i'}^+\varphi_{i'}}(p)G^{\underline{\varphi}_{i'}\underline{O}}(p,r).\tag{4.10}$$

In this way, the l.h.s. of (4.8) can be written as

$$\begin{aligned}
&\frac{\mathrm{i}}{\xi_a}q^\mu G^{[\bar{V}^a\varphi_i^+]\underline{O}}_\mu(q,p-q,r)\\
&\quad-\mathrm{i}ev\sum_{\Phi_j}I^{V^a}_{H\Phi_j}G^{[\Phi_j\varphi_i^+]\underline{O}}(q,p-q,r)\\
&\quad+\sum_{V^b}X^{V^b}_{\varphi_i^+}G^{[\bar{u}^a u^b]\underline{O}}(q,p-q,r)\\
&\quad+\sum_{\varphi_{i'}}S^{\bar{V}^a}_{\varphi_i^+\varphi_{i'}}G^{\underline{\varphi}_{i'}\underline{O}}(p,r)+\sum_{\tilde{\varphi}_j}M^{\bar{V}^a}_{\varphi_i^+\tilde{\varphi}_j}G^{\underline{\tilde{\varphi}}_j\underline{O}}(p,r),
\end{aligned}\tag{4.11}$$

where all enhanced terms are in the self-energy-like $(\varphi\varphi)$ contributions

$$\begin{aligned}
S^{\bar{V}^a}_{\varphi_i^+\varphi_{i'}} &= \frac{\mathrm{i}}{\xi_a}q^\mu G^{\bar{V}^a\varphi_i^+\varphi_{i'}}_\mu(q,p-q,-p)\\
&\quad-\mathrm{i}ev\sum_{\Phi_k}I^{V^a}_{H\Phi_k}G^{\Phi_k\varphi_i^+\varphi_{i'}}(q,p-q,-p)\\
&\quad+\sum_{V^b}X^{V^b}_{\varphi_i^+}G^{\bar{u}^a u^b\varphi_{i'}}(q,p-q,-p)\\
&\quad-\mathrm{i}eG^{\bar{u}^a u^a}(q)G^{\varphi_{i'}^+\varphi_{i'}}(p)I^{V^a}_{\varphi_{i'}\varphi_i}
\end{aligned}\tag{4.12}$$

and in the mixing-energy-like $(\varphi\tilde{\varphi})$ contributions

$$M^{\bar{V}^a}_{\varphi_i^+\tilde{\varphi}_j} = \frac{\mathrm{i}}{\xi_a}q^\mu G^{\bar{V}^a\varphi_i^+\tilde{\varphi}_j}_\mu(q,p-q,-p)$$

$$- \mathrm{i}ev \sum_{\Phi_k} I^{V^a}_{H\Phi_k} G^{\Phi_k \varphi^+_i \tilde{\varphi}_j}(q, p-q, -p) + \sum_{V^b} X^{V^b}_{\varphi^+_i} G^{\bar{u}^a u^b \tilde{\varphi}_j}(q, p-q, -p). \quad (4.13)$$

Note that here the terms originating from $\mathcal{L}_v$, i.e. terms proportional to the vev, are enhanced by the internal $\tilde{\varphi}_j$ propagators and represent leading contributions to (4.11). In order to simplify (4.12) and (4.13), and to check whether contributions proportional to the vev survive, we have to derive two further Ward identities.

(1) For the self-energy-like contributions (4.12) we exploit the BRS invariance of the Green function $\langle \bar{u}^a(x)\varphi^+_i(y) \varphi_{i'}(z)\rangle$:

$$\langle [s\bar{u}^a(x)]\varphi^+_i(y)\varphi_{i'}(z)\rangle - \langle \bar{u}^a(x)[s\varphi^+_i(y)]\varphi_{i'}(z)\rangle = \langle \bar{u}^a(x)\varphi^+_i(y)[s\varphi_{i'}(z)]\rangle. \quad (4.14)$$

Using the BRS variations (B.15), (B.13), and (B.12), we have

$$\begin{aligned} &\frac{1}{\xi_a}\partial^\mu_x \langle \bar{V}^a_\mu(x)\varphi^+_i(y)\varphi_{i'}(z)\rangle \\ &\quad - \mathrm{i}ev \sum_{\Phi_j} I^{V^a}_{H\Phi_j} \langle \Phi_j(x)\varphi^+_i(y)\varphi_{i'}(z)\rangle \\ &\quad + \sum_{V^b} X^{V^b}_{\varphi^+_i} \langle \bar{u}^a(x)u^b(y)\varphi_{i'}(z)\rangle \\ &\quad - \mathrm{i}e \sum_{V^b,\varphi_k} \langle \bar{u}^a(x)u^b(y)\varphi^+_k(y)\varphi_{i'}(z)\rangle I^{V^b}_{\varphi_k\varphi_i} \\ &= - \sum_{V^b} X^{V^b}_{\varphi_{i'}} \langle \bar{u}^a(x)\varphi^+_i(y)u^b(z)\rangle \quad (4.15) \\ &\quad - \mathrm{i}e \sum_{V^b,\varphi_k} I^{V^b}_{\varphi_{i'}\varphi_k} \langle \bar{u}^a(x)\varphi^+_i(y)u^b(z)\varphi_k(z)\rangle. \end{aligned}$$

In LO, the terms involving four fields reduce to products of pairs of propagators. After Fourier transformation we obtain

$$\begin{aligned} &\frac{\mathrm{i}}{\xi_a} q^\mu \langle \bar{V}^a_\mu(q)\varphi^+_i(p-q)\varphi_{i'}(-p)\rangle \\ &\quad - \mathrm{i}ev \sum_{\Phi_j} I^{V^a}_{H\Phi_j} \langle \Phi_j(q)\varphi^+_i(p-q)\varphi_{i'}(-p)\rangle \\ &\quad + \sum_{V^b} X^{V^b}_{\varphi^+_i} \langle \bar{u}^a(q)u^b(p-q)\varphi_{i'}(-p)\rangle \\ &\quad - \mathrm{i}e \langle \bar{u}^a(q)u^a(-q)\rangle \langle \varphi^+_{i'}(p)\varphi_{i'}(-p)\rangle I^{V^a}_{\varphi_{i'}\varphi_i} \\ &= - \sum_{V^b} X^{V^b}_{\varphi_{i'}} \langle \bar{u}^a(q)\varphi^+_i(p-q)u^b(-p)\rangle \\ &\quad - \mathrm{i}e \langle \bar{u}^a(q)u^a(-q)\rangle \\ &\quad \times \langle \varphi^+_i(p-q)\varphi_i(-p+q)\rangle I^{V^a}_{\varphi_{i'}\varphi_i}, \quad (4.16) \end{aligned}$$

and we easily see that

$$S^{\bar{V}^a}_{\varphi^+_i\varphi_{i'}} = - \sum_{V^b} X^{V^b}_{\varphi_{i'}} G^{\bar{u}^a\varphi^+_i u^b}(q, p-q, -p) - \mathrm{i}e G^{\bar{u}^a u^a}(q) G^{\varphi^+_i\varphi_i}(p-q) I^{V^a}_{\varphi_{i'}\varphi_i}. \quad (4.17)$$

(2) For the mixing-energy-like contributions (4.13) we use the BRS invariance of the Green function $\langle \bar{u}^a(x)\varphi^+_i(y) \tilde{\varphi}_j(z)\rangle$. The resulting WI is obtained from (4.16) by substituting $\varphi_{i'} \to \tilde{\varphi}_j$ and by neglecting the mixing propagators $\langle \varphi^+_i(p)\tilde{\varphi}_j(-p)\rangle$ which vanish in LO and reads

$$\begin{aligned} &\frac{\mathrm{i}}{\xi_a} q^\mu \langle \bar{V}^a_\mu(q)\varphi^+_i(p-q)\tilde{\varphi}_j(-p)\rangle \\ &\quad - \mathrm{i}ev \sum_{\Phi_k} I^{V^a}_{H\Phi_k} \langle \Phi_k(q)\varphi^+_i(p-q)\tilde{\varphi}_j(-p)\rangle \\ &\quad + \sum_{V^b} X^{V^b}_{\varphi^+_i} \langle \bar{u}^a(q)u^b(p-q)\tilde{\varphi}_j(-p)\rangle \\ &= - \sum_{V^b} X^{V^b}_{\tilde{\varphi}_j} \langle \bar{u}^a(q)\varphi^+_i(p-q)u^b(-p)\rangle. \quad (4.18) \end{aligned}$$

This relation involves the $VV\Phi$ couplings as well as other terms originating from $\mathcal{L}_v$, and leads to

$$M^{\bar{V}^a}_{\varphi^+_i\tilde{\varphi}_j} = - \sum_{V^b} X^{V^b}_{\tilde{\varphi}_j} G^{\bar{u}^a\varphi^+_i u^b}(q, p-q, -p). \quad (4.19)$$

Both (4.17) and (4.19) contain the ghost vertex function $G^{\bar{u}^a\varphi^+_i u^b}$, but when we combine them in (4.11) these ghost contributions cancel owing to the LO identity that relates external would-be Goldstone bosons and gauge bosons,

$$\begin{aligned} &\left[\sum_{V^d} G^{\underline{V^d_\mu}\,\underline{O}}(p,r) X^{V^b}_{V^d_\mu} + \sum_{\Phi_j} G^{\underline{\Phi_j}\,\underline{O}}(p,r) X^{V^b}_{\Phi_j}\right] \\ &\quad \times G^{\bar{u}^a\varphi^+_i u^b}(q, p-q, -p) \\ &= \left[-\mathrm{i}p^\mu G^{\underline{V^b_\mu}\,\underline{O}}(p,r) + \mathrm{i}ev \sum_{\Phi_j} I^{V^b}_{\Phi_j H} G^{\underline{\Phi_j}\,\underline{O}}(p,r)\right] \\ &\quad \times G^{\bar{u}^a\varphi^+_i u^b}(q, p-q, -p) = 0. \quad (4.20) \end{aligned}$$

Thus, all mixing terms cancel, and the complete identity (4.8) becomes

$$\begin{aligned} &\frac{\mathrm{i}}{\xi_a} q^\mu G^{[\bar{V}^a\varphi^+_i]\underline{O}}_\mu(q, p-q, r) \\ &\quad - \mathrm{i}ev \sum_{\Phi_j} I^{V^a}_{H\Phi_j} G^{[\Phi_j\varphi^+_i]\underline{O}}(q, p-q, r) \\ &\quad + \sum_{V^b} X^{V^b}_{\varphi^+_i} G^{[\bar{u}^a u^b]\underline{O}}(q, p-q, r) \\ &\quad - \mathrm{i}e G^{\bar{u}^a u^a}(q) G^{\varphi^+_i\varphi_i}(p-q) \sum_{\varphi_{i'}} G^{\underline{\varphi_{i'}}\,\underline{O}}(p,r) I^{V^a}_{\varphi_{i'}\varphi_i} \\ &= \mathrm{i}e \sum_{V^b,\varphi_{i'}} \sum_{O_1 \neq O} G^{\underline{\varphi^+_{i'}}\,\underline{O}_1}(p+r_2, r_1) I^{V^b}_{\varphi_{i'}\varphi_i} \\ &\quad \times G^{\bar{u}^a u^b \underline{O}_2}(q, -q-r_2, r_2). \quad (4.21) \end{aligned}$$

Now we can truncate the two remaining external legs. To this end we observe that (see Appendix C) the longitudinal part of the LO gauge-boson propagator $G^{V^a\bar{V}^a}_{\mathrm{L}}(q)$, the LO ghost propagator, and the LO propagator of the associated would-be Goldstone boson Φ_j are related by

$$\frac{1}{\xi_a} G_{\mathrm{L}}^{V^a \bar{V}^a}(q) = G^{\bar{u}^a u^a}(q) = -G^{\Phi_j^+ \Phi_j}(q). \tag{4.22}$$

Using this relation, the leg with momentum q is easily truncated by multiplying the above identity with the longitudinal part of the inverse gauge-boson propagator $-\mathrm{i}\xi_a \Gamma_{\mathrm{L}}^{V^a \bar{V}^a}(q)$. The leg with momentum $p-q$ is truncated by multiplying (4.21) by the inverse (scalar-boson or gauge-boson) propagator $-\mathrm{i}\Gamma^{\varphi_i \varphi_i^+}(p-q)$, and by using

$$X_{\varphi_i^+}^{V^b} G^{u^b \bar{u}^b}(p-q) = c_{\varphi_i} G^{\varphi_i^+ \varphi_i}(p-q) X_{\varphi_i^+}^{V^b} \tag{4.23}$$

with $c_{\Phi_i} = 1$ and $c_{V^b} = -1/\xi_b$. The truncated identity reads

$$\begin{aligned}
&\mathrm{i}q^\mu G_\mu^{[\underline{V^a} \underline{\varphi_i}]\underline{O}}(q, p-q, r) \\
&\quad + \mathrm{i}ev \sum_{\Phi_j} I_{H\Phi_j}^{V^a} G^{[\underline{\Phi_j^+} \underline{\varphi_i}]\underline{O}}(q, p-q, r) \\
&\quad + \sum_{V^b} c_{\varphi_i} X_{\varphi_i^+}^{V^b} G^{[\underline{u^a} \underline{\bar{u}^b}]\underline{O}}(q, p-q, r) \\
&\quad - \mathrm{i}e \sum_{\varphi_{i'}} G^{\underline{\varphi_{i'}} \underline{O}}(p, r) I_{\varphi_{i'} \varphi_i}^{V^a} \\
&= e \sum_{V^b, \varphi_{i'}} \sum_{O_1 \neq O} \left[\Gamma^{\varphi_i \varphi_i^+}(p-q) G^{\varphi_{i'}^+ \underline{O_1}}(-r_1, r_1)\right] \\
&\qquad \times I_{\varphi_{i'} \varphi_i}^{V^b} G^{\underline{u^a} u^b \underline{O_2}}(q, -q-r_2, r_2).
\end{aligned} \tag{4.24}$$

Now we contract with the wave function $v_\varphi(p)$ of an on-shell external state with mass $(p^2)^{1/2}$. For scalar bosons the wave function is trivial ($v_\Phi(p) = 1$), whereas for external gauge bosons we consider transverse polarizations $v_{V_\nu}(p) = \varepsilon_{\mathrm{T}}^\nu(p)$. Finally, when we take the collinear limit $q^\mu \to xp^\mu$ and assume

$$M^2 \sim \max(p^2, M_{\varphi_i}^2, M_{V^a}^2) \ll s, \tag{4.25}$$

various terms in (4.24) are mass-suppressed. The r.h.s. is mass-suppressed owing to

$$\begin{aligned}
&\lim_{q^\mu \to xp^\mu} v_\varphi(p) \Gamma^{\varphi_i \varphi_i^+}(p-q) G^{\varphi_{i'}^+ \varphi_{i'}}(-r_1) \\
&\quad \sim \lim_{q^\mu \to xp^\mu} \frac{(p-q)^2 - M_{\varphi_i}^2}{r_1^2} = \mathcal{O}\left(\frac{M^2}{s}\right),
\end{aligned} \tag{4.26}$$

since r_1 is a non-trivial combination of the external momenta, and like for all invariants we assume that $r_1^2 \sim s$, whereas in the collinear limit $(p-q)^2 - M_{\varphi_i}^2 \sim M^2$. The second term on the l.h.s. of (4.24) is proportional to the vev and therefore mass-suppressed, and for the third term we have

$$\lim_{q^\mu \to xp^\mu} v_\varphi(p) X_{\varphi_i^+}^{V^b} = \mathcal{O}(M). \tag{4.27}$$

For gauge bosons this is due to the transversality of the polarization vector

$$\lim_{q^\mu \to xp^\mu} (p-q)_\nu \varepsilon_{\mathrm{T}}^\nu(p) = 0, \tag{4.28}$$

whereas for scalar bosons $X_{\Phi_i^+}^{V^b}$ is explicitly proportional to the vev. The remaining leading terms give the result

$$\begin{aligned}
&\lim_{q^\mu \to xp^\mu} q^\mu v_\varphi(p) G_\mu^{[\underline{V^a} \underline{\varphi_i}]\underline{O}}(q, p-q, r) \\
&\quad = e \sum_{\varphi_{i'}} v_\varphi(p) G^{\underline{\varphi_{i'}} \underline{O}}(p, r) I_{\varphi_{i'} \varphi_i}^{V^a} + \mathcal{O}\left(\frac{M^2}{s} \mathcal{M}^{\varphi_i O}\right) \\
&\quad = e \sum_{\varphi_{i'}} \mathcal{M}^{\varphi_{i'} O}(p, r) I_{\varphi_{i'} \varphi_i}^{V^a} + \mathcal{O}\left(\frac{M^2}{s} \mathcal{M}^{\varphi_i O}\right),
\end{aligned} \tag{4.29}$$

which is the identity represented in (4.2) in diagrammatic form. Note that in general the mass of the wave function $v_\varphi(p)$ need not be equal to the masses of the fields φ_i or $\varphi_{i'}$.

4.2 Fermions

The derivation of the collinear Ward identity for external fermions $\varphi_i = \Psi_{j,\sigma}^\kappa$ is completely analogous to that presented in the previous section. In fact, it is much simpler since no mixing contributions ($\tilde{\varphi}$) have to be considered. The effect of the quark-mixing matrix can be absorbed in the generalized generators

$$I_{\varphi_i \varphi_{i'}}^{V^a} = U_{jj'}^{V^a} I_{\sigma\sigma'}^{V^a}. \tag{4.30}$$

The final result reads

$$\begin{aligned}
&\lim_{q^\mu \to xp^\mu} q^\mu G_\mu^{[\underline{V^a} \underline{\Psi_{j,\sigma}^\kappa}]\underline{O}}(q, p-q, r) u(p) \\
&\quad = +e \sum_{\sigma', j'} \mathcal{M}^{f_{j',\sigma'}^\kappa O}(p, r) I_{\sigma'\sigma}^{V^a} U_{j'j}^{V^a} + \mathcal{O}\left(\frac{M}{\sqrt{s}} \mathcal{M}^{f_{j,\sigma}^\kappa O}\right),
\end{aligned} \tag{4.31}$$

for fermions, and

$$\begin{aligned}
&\lim_{q^\mu \to xp^\mu} q^\mu \bar{v}(p) G_\mu^{[\underline{V^a} \underline{\bar{\Psi}_{j,\sigma}^\kappa}]\underline{O}}(q, p-q, r) \\
&\quad = -e \sum_{\sigma', j'} I_{\sigma\sigma'}^{V^a} U_{jj'}^{V^a} \mathcal{M}^{\bar{f}_{j',\sigma'}^\kappa O}(p, r) + \mathcal{O}\left(\frac{M}{\sqrt{s}} \mathcal{M}^{\bar{f}_{j,\sigma}^\kappa O}\right),
\end{aligned} \tag{4.32}$$

for antifermions, where

$$M^2 \sim \max(p^2, m_{f_{j,\sigma}}^2, M_{V^a}^2). \tag{4.33}$$

In the derivation we used

$$\begin{aligned}
&\lim_{q^\mu \to xp^\mu} \bar{v}(p) \Gamma_{j,\sigma}^{\bar{\Psi}\Psi}(p-q) G_{j',\sigma'}^{\Psi\bar{\Psi}}(-r_1) \\
&\quad \propto \frac{M \bar{v}(p) \not{r}_1}{r_1^2} = \mathcal{O}\left(\frac{M}{\sqrt{s}}\right),
\end{aligned} \tag{4.34}$$

which is the analogue of (4.26), and a similar equation for fermion spinors $u(p)$.

5 Conclusions

For energies at and beyond 1 TeV, the electroweak corrections are dominated by double and single logarithms involving the ratio of the typical energy of the considered process to the electroweak scale. For processes that are not mass-suppressed, the one-loop logarithmic corrections are universal, i.e. in contrast to the non-logarithmic corrections they can be calculated in a process-independent way. The corresponding results have already been published in [7].

Here we have presented the derivation of the virtual collinear logarithms at the one-loop level in the electroweak standard model for processes that are not mass-suppressed. Using the BRS invariance of the electroweak standard model, we have proved the factorization of these logarithms in the 't Hooft–Feynman gauge. The proof has been performed in the spontaneously broken phase in terms of the physical fields and parameters. The mixings between the various fields and all relevant terms proportional to the vacuum expectation value have been taken into account. We find that all terms proportional to the vacuum expectation value cancel and the results are equivalent to those obtained in the symmetric phase with the longitudinal modes of the gauge bosons replaced by the would-be Goldstone bosons as physical particles. Thus, this equivalence, which has been assumed in the literature, has been proven at the one-loop level using the Goldstone-boson equivalence theorem and the corresponding corrections. It will be interesting to investigate to what extent this equivalence is valid at higher orders.

While we have derived the collinear Ward identities and the collinear logarithms within the electroweak standard model our method can be generalized to arbitrary spontaneously broken gauge theories including, in particular, supersymmetric extensions of the standard model.

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Appendix

A Collinear singularity

In this appendix we discuss mass singularities originating from integrals of the type

$$I = -\mathrm{i}(4\pi)^2\mu^{4-D} \times \int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{N(q)}{(q^2-M_0^2+\mathrm{i}\varepsilon)[(p-q)^2-M_1^2+\mathrm{i}\varepsilon]}. \tag{A.1}$$

We restrict ourselves to purely collinear singularities that are exclusively related to an external relativistic momentum p^μ that has a small square, i.e. $p^2 \ll (p^0)^2 \sim s$. Singularities originating from other propagators in $N(q)$ are not considered. In particular, we assume that $N(q)$ is either not singular in the soft limit $q^\mu \to 0$ or that the soft singularities are subtracted.

Our goal is to fix a precise prescription for extracting the part of the function $N(q)$ that enters the mass-singular part of (A.1). To this end, we introduce a Sudakov parametrization [15] for the loop momentum

$$q^\mu = xp^\mu + yl^\mu + q_\mathrm{T}^\mu, \tag{A.2}$$

where p^μ and the light-like four vector l^μ,

$$p^\mu = (p^0, \boldsymbol{p}), \quad l^\mu = (p^0, -p^0\boldsymbol{p}/|\boldsymbol{p}|), \tag{A.3}$$

describe the component collinear to the external momentum, whereas the space-like vector q_T^μ with

$$p_\mu q_\mathrm{T}^\mu = l_\mu q_\mathrm{T}^\mu = 0, \quad q_\mathrm{T}^2 = -|\boldsymbol{q}_\mathrm{T}|^2 \tag{A.4}$$

represents the perpendicular component. In this parametrization we get

$$I = -4\mathrm{i}(pl)\mu^{4-D}\int \mathrm{d}x \int \mathrm{d}y \int \frac{\mathrm{d}^{D-2}q_\mathrm{T}}{(2\pi)^{D-2}} \frac{N(q)}{(q^2-M_0^2+\mathrm{i}\varepsilon)[(p-q)^2-M_1^2+\mathrm{i}\varepsilon]}. \tag{A.5}$$

The denominators of the propagators read

$$\begin{aligned} q^2-M_0^2+\mathrm{i}\varepsilon &= x^2p^2+2xy(pl) \\ &\quad -|\boldsymbol{q}_\mathrm{T}|^2-M_0^2+\mathrm{i}\varepsilon, \\ (p-q)^2-M_1^2+\mathrm{i}\varepsilon &= (1-x)^2p^2+2(x-1)y(pl) \\ &\quad -|\boldsymbol{q}_\mathrm{T}|^2-M_1^2+\mathrm{i}\varepsilon, \end{aligned} \tag{A.6}$$

and are linear in the variable y owing to $l^2=0$. For $x \neq 0, 1$, the integral I can be written as

$$I = -\mathrm{i}\frac{\mu^{4-D}}{(pl)}\int \frac{\mathrm{d}x}{x(x-1)}\int \frac{\mathrm{d}^{D-2}q_\mathrm{T}}{(2\pi)^{D-2}} \int \mathrm{d}y \frac{N(x,y,q_\mathrm{T})}{(y-y_0)(y-y_1)}, \tag{A.7}$$

with single poles at

$$\begin{aligned} y_0 &= \frac{|\boldsymbol{q}_\mathrm{T}|^2-x^2p^2+M_0^2-\mathrm{i}\varepsilon}{2x(pl)}, \quad x\neq 0, \\ y_1 &= \frac{|\boldsymbol{q}_\mathrm{T}|^2-(1-x)^2p^2+M_1^2-\mathrm{i}\varepsilon}{2(x-1)(pl)}, \quad x \neq 1. \end{aligned} \tag{A.8}$$

The y integral is non-zero only when the poles lie in opposite complex half-planes, i.e. for $0<x<1$. Then, it can be performed by closing the contour around one of the two poles. This yields

$$\begin{aligned} I &= -\frac{\mu^{4-D}}{(pl)}\int_0^1 \frac{\mathrm{d}x}{x(x-1)}\int \frac{\mathrm{d}^{D-2}q_\mathrm{T}}{(2\pi)^{D-3}}\frac{N(x,y_i,q_\mathrm{T})}{y_0-y_1} \\ &= 4\pi\mu^{4-D}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^{D-2}q_\mathrm{T}}{(2\pi)^{D-2}}\frac{N(x,y_i,q_\mathrm{T})}{|\boldsymbol{q}_\mathrm{T}|^2+\Delta(x)}, \end{aligned} \tag{A.9}$$

where in the vicinity of $x=1,0$ the contour has to be closed around the pole at $y_i=y_0,y_1$, respectively.

The transverse momentum integral exhibits a logarithmic singularity in the collinear region $|q_\mathrm{T}| \to 0$, where the squares of the momenta p and $p-q$ are small compared to the energy squared $p^2, (p-q)^2 \ll (pl) \sim 2p_0^2$. The singularity is regulated by the mass terms in

$$\Delta(x) = (1-x)M_0^2 + xM_1^2 - x(1-x)p^2. \qquad \text{(A.10)}$$

In leading approximation, we restrict ourselves to logarithmic mass-singular contributions in (A.9). Terms of order $|\boldsymbol{q}_\mathrm{T}|^2$, p^2, M_0 or M_1 are neglected in $N(q)$. Since the relevant pole, y_0 or y_1, is of order $|\boldsymbol{q}_\mathrm{T}|^2/(pl)$, also contributions proportional to y can be discarded. We therefore arrive at the following simple recipe for $N(q)$ in the collinear limit:

(1) Substitute $N(x,y,q_\mathrm{T}) \to N(x,0,0)$, i.e. replace $q^\mu \to xp^\mu$. (A.11)

(2) Neglect all mass-suppressed contributions.

Then, performing the q_T integration in $D-2 = 2-2\varepsilon$ dimensions and expanding in ε, we obtain the leading contribution

$$\begin{aligned} I &= \Gamma(\varepsilon)\int_0^1 \mathrm{d}x \left(\frac{4\pi\mu^2}{\Delta(x)}\right)^\varepsilon N(x,0,0) \\ &= \frac{1}{\varepsilon} + \int_0^1 \mathrm{d}x \log\left(\frac{\mu^2}{\Delta(x)}\right) N(x,0,0) \\ &\quad - \gamma + \log 4\pi + \mathcal{O}(\varepsilon). \end{aligned} \qquad \text{(A.12)}$$

Finally, omitting the UV singularity, which cancels in observables, neglecting constant terms, and performing the integral, we obtain

$$I \overset{\mathrm{LA}}{=} \log\left(\frac{\mu^2}{M^2}\right)\int_0^1 \mathrm{d}x\, N(x,0,0), \qquad \text{(A.13)}$$

in logarithmic approximation (LA). The scale in the logarithm is of the order of the largest mass in (A.10),

$$M^2 \sim \max(p^2, M_0^2, M_1^2). \qquad \text{(A.14)}$$

B BRS transformations

In this appendix we summarize our conventions for the gauge-fixing terms and the BRS symmetry of the electroweak standard model. We follow [14] but introduce a generic notation.

B.1 Gauge symmetry

The classical Lagrangian of the electroweak standard model is invariant with respect to gauge transformations of the physical fields (and would-be Goldstone bosons) φ_i, which can generically be written as

$$\delta\varphi_i(x) = \sum_{V^b=A,Z,W^\pm}\left[X_{\varphi_i}^{V^b}\delta\theta^{V^b}(x) + \mathrm{i}e\sum_{\varphi_{i'}} I_{\varphi_i\varphi_{i'}}^{V^b}\delta\theta^{V^b}(x)\varphi_{i'}(x)\right]. \qquad \text{(B.1)}$$

The linear operator $X_{\varphi_i}^{V^b}$ represents the transformation of free fields, and the non-linear term contains the $\mathrm{SU(2)}\times\mathrm{U(1)}$ generators $I_{\varphi_i\varphi_{i'}}^{V^b}$ in the representation of the fields φ_i [7]. For scalar bosons, the linear term in (B.1) is determined by the contribution of the vev

$$\mathbf{v}_i = v\delta_{H\Phi_i} \qquad \text{(B.2)}$$

and reads

$$X_{\Phi_i}^{V^b}\delta\theta^{V^b}(x) = \mathrm{i}ev I_{\Phi_i H}^{V^b}\delta\theta^{V^b}(x). \qquad \text{(B.3)}$$

For gauge bosons, $\varphi_i = V^b = A, Z, W^\pm$, we have

$$X_{V_\mu^c}^{V^b}\delta\theta^{V^b}(x) = \delta_{V^bV^c}\partial_\mu\delta\theta^{V^b}(x), \qquad \text{(B.4)}$$

which in momentum space leads to the simple relation

$$X_{V_\mu^c}^{V^b}\delta\theta^{V^b}(p) = \mathrm{i}p_\mu\delta_{V^bV^c}\delta\theta^{V^b}(p). \qquad \text{(B.5)}$$

For fermions, $\varphi_i = \Psi_{j,\sigma}^\kappa$,

$$X_{\Psi_{j,\sigma}}^{V^b}\delta\theta^{V^b}(x) = 0, \qquad \text{(B.6)}$$

and the gauge transformation of the physical fields is determined by

$$I_{\Psi_{j,\sigma}\Psi_{j',\sigma,}}^{V^a} = U_{jj'}^{V^a} I_{\sigma\sigma'}^{V^a}, \qquad \text{(B.7)}$$

where the generators $I_{\sigma\sigma'}^{V^a}$ depend on the representation of $\Psi_{j,\sigma}^\kappa$ and, in particular, on the chirality $\kappa = \mathrm{R}, \mathrm{L}$. The unitary mixing matrix $U_{jj'}^{V^a}$ is trivial ($U_{jj'}^{V^a} = \delta_{jj'}$) everywhere except for the left-handed quark representation, where it has the non-trivial components

$$U_{jj'}^{W^+} = \mathbf{V}_{jj'}, \quad U_{jj'}^{W^-} = \mathbf{V}_{jj'}^+ = \mathbf{V}_{j'j}^*, \qquad \text{(B.8)}$$

corresponding to the quark-mixing matrix $\mathbf{V}_{jj'}$.

B.2 Gauge fixing and BRS invariance

The quantized electroweak Lagrangian includes the gauge-fixing term

$$\mathcal{L}_\mathrm{fix} = -\sum_{V^a=A,Z,W^\pm}\frac{1}{2\xi_a}C^{V^a}C^{\bar{V}^a}, \qquad \text{(B.9)}$$

with the gauge parameters ξ_A, ξ_Z, $\xi_+ = \xi_-$, and the corresponding ghost terms. The charge-conjugate of V is denoted $\bar{V}$. A general 't Hooft gauge fixing is given by

$$C^{\bar{V}^a}\{V,\Phi,x\} = \partial^\mu\bar{V}_\mu^a(x) - \mathrm{i}ev\xi_a\sum_{\Phi_i=H,\chi,\phi^\pm} I_{H\Phi_i}^{V^a}\Phi_i(x). \qquad \text{(B.10)}$$

Note that the matrix elements $I_{H\Phi_i}^{V^a}$ relate the gauge fields V^a to the associated would-be Goldstone-boson fields Φ_i. In fact, the single components of (B.10) read

$$\begin{aligned} C^A(x) &= \partial^\mu A_\mu(x), \\ C^Z(x) &= \partial^\mu Z_\mu(x) - \xi_Z M_Z\chi(x), \\ C^\pm(x) &= \partial^\mu W_\mu^\pm(x) \mp \mathrm{i}\xi_\pm M_W\phi^\pm. \end{aligned} \qquad \text{(B.11)}$$

In the 't Hooft gauge the contributions of the would-be Goldstone bosons to the gauge-fixing terms cancel the LO mixing between gauge bosons and would-be Goldstone bosons.

The gauge-fixing terms and the ghost terms break the gauge invariance of the classical electroweak Lagrangian. However, the complete electroweak Lagrangian is invariant with respect to BRS transformations of the ghost and physical fields.

The BRS transformation of the physical fields corresponds to a local gauge transformation (B.1) with gauge-transformation parameters $\delta\theta^{V^a}(x) = \delta\lambda u^a(x)$ determined by the ghost fields $u^a(x)$ and the infinitesimal Grassmann parameter $\delta\lambda$. To be precise, the BRS variation $s\varphi_i(x)$ is defined as left derivative[4] with respect to the Grassmann parameter $\delta\lambda$, i.e. $\delta\varphi_i(x) = \delta\lambda s\varphi_i(x)$, and reads

$$s\varphi_i(x) = \sum_{V^b=A,Z,W^\pm} \left[X^{V^b}_{\varphi_i} u^b(x) + \mathrm{i}e \sum_{\varphi_{i'}} I^{V^b}_{\varphi_i\varphi_{i'}} u^b(x)\varphi_{i'}(x) \right]. \qquad \text{(B.12)}$$

The BRS variation for charge-conjugate fields is obtained from the adjoint of (B.12) as

$$s\varphi_i^+(x) = \sum_{V^b=A,Z,W^\pm} \left[X^{V^b}_{\varphi_i^+} u^b(x) - \mathrm{i}e \sum_{\varphi_{i'}} u^b(x)\varphi^+_{i'}(x) I^{V^b}_{\varphi_{i'}\varphi_i} \right], \qquad \text{(B.13)}$$

where we have used $\left(I^{V^a}\right)^+ = I^{\bar{V}^a}$.

The BRS variation of the ghost fields is given by

$$su^b(x) = \frac{\mathrm{i}e}{2} \sum_{V^a,V^c=A,Z,W^\pm} I^{V^a}_{V^bV^c} u^a(x) u^c(x). \qquad \text{(B.14)}$$

The BRS variation of the antighost fields is determined by the gauge-fixing terms,

$$s\bar{u}^a(x) = -\frac{1}{\xi_a} C^{\bar{V}^a}\{V,\Phi,x\}. \qquad \text{(B.15)}$$

C Conventions for Green functions

Our conventions for Green functions are based on [14]. In configuration space we use the equivalent notations

$$G^{\varphi_{i_1}\cdots\varphi_{i_n}}(x_1,\ldots,x_n) = \langle \varphi_{i_1}(x_1)\ldots\varphi_{i_n}(x_n)\rangle. \qquad \text{(C.1)}$$

Fourier transformation is defined with incoming momenta, and the momentum-conservation δ-function is factorized as

$$(2\pi)^4\delta^{(4)}\left(\sum_{k=1}^n p_k\right) G^{\varphi_{i_1}\cdots\varphi_{i_n}}(p_1,\ldots,p_n) = \int \left(\prod_{k=1}^n \mathrm{d}^4x_k\right) \exp\left(-\mathrm{i}\sum_{j=1}^n x_jp_j\right) \times G^{\varphi_{i_1}\cdots\varphi_{i_n}}(x_1,\ldots,x_n). \qquad \text{(C.2)}$$

Because the field operator φ creates antiparticles and annihilates particles, the fields in the Green functions are associated with outgoing particles (incoming antiparticles). For propagators we introduce the shorthand notation

$$G^{\varphi_i\varphi_j}(p) = G^{\varphi_i\varphi_j}(p,-p), \qquad \text{(C.3)}$$

For the truncation of the external leg φ_{i_k} in momentum space we adopt the convention

$$G^{\cdots\varphi_{i_k}\cdots}(\ldots,p_k,\ldots) = G^{\varphi_{i_k}\varphi^+_{i_k}}(p_k) G^{\cdots\underline{\varphi^+_{i_k}}\cdots}(\ldots,p_k,\ldots), \qquad \text{(C.4)}$$

where the field argument corresponding to the truncated leg is underlined and where we have assumed diagonal propagators. In truncated Green functions the fields are associated with incoming particles.

The (diagonal) propagators are related to the 2-point vertex functions by

$$G^{\varphi_i\varphi_i^+}(p,-p)\Gamma^{\varphi_i^+\varphi_i}(p,-p) = \pm\mathrm{i}, \qquad \text{(C.5)}$$

with + for scalars and gauge bosons and − for fermions and ghosts.

In the 't Hooft gauge, the LO propagators are diagonal. They read

$$G^{V^a\bar{V}^b}_{\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) G^{V^a\bar{V}^b}_{\mathrm{T}}(p) + \frac{p_\mu p_\nu}{p^2} G^{V^a\bar{V}^b}_{\mathrm{L}}(p), \qquad \text{(C.6)}$$

with

$$G^{V^a\bar{V}^b}_{\mathrm{T}}(p) = \frac{-\mathrm{i}\delta_{V^aV^b}}{p^2 - M^2_{V^a}}, \quad G^{V^a\bar{V}^b}_{\mathrm{L}}(p) = \frac{-\mathrm{i}\xi_a\delta_{V^aV^b}}{p^2 - \xi_a M^2_{V^a}} \qquad \text{(C.7)}$$

for gauge bosons and

$$G^{HH}(p) = \frac{\mathrm{i}}{p^2 - M_H^2}, \quad G^{\Phi_a^+\Phi_b}(p) = \frac{\mathrm{i}\delta_{V^aV^b}}{p^2 - \xi_a M^2_{V_a}} \qquad \text{(C.8)}$$

for Higgs bosons and would-be Goldstone bosons $\Phi_a = \chi, \phi^\pm$ associated to the weak gauge bosons $V^a = Z, W^\pm$. The propagators for ghost fields are given by

$$G^{u^a\bar{u}^b}(-p) = -G^{\bar{u}^bu^a}(p) = \frac{\mathrm{i}\delta_{V^aV^b}}{p^2 - \xi_a M^2_{V^a}}, \qquad \text{(C.9)}$$

[4] The product rule for a Grassmann left derivative reads $s(\varphi_i\varphi_j) = (s\varphi_i)\varphi_j + (-1)^{n(\varphi_i)}\varphi_i s\varphi_j$, where $n(\varphi_i)$ is given by the ghost plus the fermion number of the field φ_i

and care must be taken for the sign resulting from the anticommutativity of the ghost fields. Similarly, for fermionic fields we have

$$G^{\Psi_\alpha \bar{\Psi}_\beta}(-p) = -G^{\bar{\Psi}_\beta \Psi_\alpha}(p) = \frac{\mathrm{i}(\not{p} + m)_{\alpha\beta}}{p^2 - m^2}, \qquad \text{(C.10)}$$

where α, β are Dirac indices.

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